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# Interairline Equity in Airport Scheduling Interventions

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**Abstract.** In the absence of opportunities for capacity expansion or operational enhancements, air traffic congestion mitigation may require scheduling interventions to control overcapacity scheduling at busy airports. Previous research has shown that large delay reductions could be achieved through comparatively small changes in the schedule of flights. While existing approaches have focused on minimizing the overall impact across the airlines, this paper designs, optimizes, and assesses a novel approach for airport scheduling interventions that incorporates interairline equity objectives. It relies on a multilevel modeling architecture based on on-time performance (i.e., mitigating airport congestion), efficiency (i.e., meeting airline scheduling preferences), and equity (i.e., balancing scheduling adjustments fairly among the airlines) objectives, subject to scheduling and network connectivity constraints. Theoretical results show that, under some scheduling conditions, equity and efficiency can be jointly maximized. Computational results suggest that, under a wide range of current and hypothetical scheduling settings, ignoring interairline equity can lead to highly inequitable outcomes, but that our modeling approach achieves interairline equity at no, or small, losses in efficiency.

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**Keywords:** airport demand management • interairline equity • efficiency-equity trade-off • integer programming

## 1. Introduction

The development of air transportation systems worldwide has been supported by airport and air traffic management infrastructure. However, limitations on infrastructure capacity, coupled with significant growth in air traffic, have resulted in severe congestion at many of the world's busiest airports. This congestion typically materializes in the form of flight delays and cancellations. The costs of air traffic congestion in the United States was estimated at over $30 billion for the year 2007 (Ball et al. 2010) and this issue is likely to become even more pressing over the medium- and long-term horizons as demand for air traffic is expected to increase nationally and internationally.

Most of the air traffic delays in the United States originate from imbalances between demand and capacity at the busiest airports (Bureau of Transportation Statistics 2013). In the absence of opportunities for capacity expansion or operating enhancements, such imbalances can only be significantly mitigated through scheduling interventions that limit the extent of overcapacity scheduling at peak hours. This paper proposes and evaluates a quantitative approach to optimize such interventions in a way that achieves on-time performance objectives, while minimizing interference with airlines' competitive scheduling and, for the first time, balancing the impact of such interventions equitably among the airlines. Before presenting the contributions of this paper (Section 1.3), we elaborate on the approaches for airport scheduling interventions (Section 1.1) and review existing work on the trade-off between efficiency and equity in resource allocation (Section 1.2).

### 1.1. Airport Scheduling Interventions

Scheduling interventions refer to the demand management measures that impose limits, or constraints, on the number of flights scheduled at an airport. They are implemented months in advance of the day of operations (before flight schedules get published and flight tickets get marketed to passengers). Most airports outside the United States operate under slot control policies that limit the number of flights scheduled per hour (or other units of time) and distribute a corresponding number of slots across the different airlines through an administrative procedure (International Air Transport Association 2015). By contrast, no demand management is applied at a large majority of U.S. airports. A few of the busiest airports were subject to slot restrictions under the High Density Rule, but, since its phaseout in 2007, airline schedules of flights in the United States have been subject to limited constraints. Given the high delays that ensued in 2007, "flight caps" have been imposed at the three major airports in the New York Metroplex, but they were found too high to effectively alleviate congestion (Office of Inspector General 2010; Government Accountability Office 2012; de Neufville and Odoni 2013). Given these regulatory differences, European airports may reject flight requests that could be accommodated, resulting in smaller throughput than their U.S. counterparts. On the other hand, U.S. airports face larger imbalances between demand and capacity and hence, larger and less predictable delays (Morisset and Odoni 2011; Odoni et al. 2011).

Recent research has showed the potential to improve current scheduling intervention practices to mitigate congestion and satisfy airline scheduling requests as closely as possible. First, market mechanisms based on congestion pricing (Carlin and Park 1970; Daniel 1995; Brueckner 2002; Vaze and Barnhart 2012a) or slot auctions (Rassenti, Smith, and Bulfin 1982; Ball, Donohue, and Hoffman 2006; Harsha 2009) have been proposed to allocate airport capacity to the users that assign the highest value to it. However, they have not been successfully implemented in the current institutional environment, most likely because of the monetary transfers they involve. A second line of research has focused on improving current slot allocation procedures at slot-controlled airports by optimizing the matching of airlines' scheduling requests (Zografos, Salouras, and Madas 2012; Zografos, Madas, and Androutsopoulos 2017). Third, several studies have investigated the potential of demand management to mitigate congestion at U.S. airports, based on the well-known result from queuing theory that the relationship between air traffic demand, airport capacity, and on-time performance is highly nonlinear at airports operating close to capacity (de Neufville and Odoni 2013; Pyrgiotis, Malone, and Odoni 2013; Nikoleris and Hansen 2012; Jacquillat and Odoni 2015b). Using a game-theoretic framework of airline frequency competition, Vaze and Barnhart (2012b) showed that small reductions in allocated airport capacity can reduce delays and improve airline profitability. By modeling the trade-off between flight delays and passenger schedule delay (i.e., schedule inconvenience), Swaroop et al. (2012) found that a reduction in allocated capacity of 10% to 20% would improve passenger welfare at a majority of busy U.S. airports. Using a different approach, Pyrgiotis and Odoni (2016) and Jacquillat and Odoni (2015a) modeled and optimized the intraday scheduling interventions, and found that limited changes in airline timetabling of flights could yield significant delay reductions. In summary, evidence suggests that performance improvements could be achieved at the busiest U.S. airports through limited scheduling interventions that involve only temporal shifts in demand (i.e., changes in the intraday timetabling of flights), and no reduction in overall demand (i.e., no change in the set of flights scheduled over the day).

These existing approaches suffer from two main limitations. First, they are focused exclusively on overall scheduling levels at the airports, without considering explicitly the impact of the interventions on the different airlines. In turn, they may penalize one airline (or a small number of airlines) disproportionately. Second, they do not investigate the potential for strategic behaviors from the airlines when providing their scheduling inputs, that is, the potential incentives for the airlines to deviate from their true preferences while providing their inputs to gain a strategic advantage over their competitors through the scheduling interventions. This paper addresses the first of these concerns by integrating interairline equity considerations into the decision-making framework underlying airport scheduling interventions. We leave the second one for future research.

Our scheduling process uses, as a starting point, capacity estimates at an airport under consideration, and the preferred schedule of flights. Airport capacity estimates can be obtained from historical records of operations (Gilbo 1993; Simaiakis 2012). As in current practice, the preferred schedule is typically provided by the airlines to a central decision maker (e.g., administratively appointed schedule coordinators at slot-controlled airports, the Federal Aviation Administration (FAA) in the United States), who then produces a schedule of flights to reduce anticipated delays at the considered airport. Per the discussion above, we focus primarily on the case where these adjustments involve only temporal shifts in demand. We also discuss the case where the adjustments involve reductions in demand (i.e., the elimination of some flights), as may be required at a few of the busiest slot-controlled airports worldwide, where unconstrained airline demand may be so high that acceptable delay levels cannot be attained with existing levels of capacity. In addition, we consider the general case where each flight is assigned a weight characterizing the cost, or the inconvenience, of this flight being rescheduled (or eliminated). This captures the standard "a flight is a flight" paradigm (with equal weights assigned to all flights, as currently practiced at slot-controlled airports and assumed in other previously proposed mechanisms), as well as extensions of existing mechanisms in which the airlines can signal the relative rescheduling costs of different flights through nonmonetary credit allocation or a monetary auction-based mechanism. In turn, this paper introduces interairline equity in a wide range of settings representing current practice as well as potential extensions of previously proposed mechanisms.

### 1.2. Equity in Resource Allocation

Airport scheduling interventions fall into a broader class of problems involving the allocation of scarce resources by a central decision maker to distributed agents. One major challenge in these problems involves defining the objective of resource allocation to balance the preferences and requirements of various stakeholders. This may create trade-offs between efficiency (i.e., maximizing the sum of agents' utilities), equity (i.e., balancing utilities fairly among the agents), and, possibly, other objectives (e.g., maximizing outcome predictability, ensuring incentive compatibility, etc.). The trade-off between efficiency and equity was first studied by Nash (1950) and Kalai and Smorodinsky (1975) for the two-player bargaining problem. It has been extended by Bertsimas, Farias, and Trichakis (2011, 2012) to general resource allocation problems, who obtained theoretical bounds on the "price of fairness" and the "price of efficiency," that is, the relative loss in efficiency if equity is maximized, and vice versa. These bounds were derived with general utility functions, and in special instances involving compact and convex utility sets. However, no study to date has incorporated interairline equity considerations into the design of airport scheduling interventions.

In a related area, equitable approaches have been developed for allocating aviation capacity on the day of operations through Air Traffic Flow Management (ATFM) initiatives. ATFM consists of optimizing the flows of aircraft at airports or through air traffic control sectors to reduce local imbalances between demand and capacity. Whereas early developments were exclusively based on efficiency objectives (minimizing congestion costs), recent studies have incorporated interairline equity considerations into the objective of ATFM models (Vossen et al. 2003; Vossen and Ball 2006; Barnhart et al. 2012; Bertsimas and Gupta 2016; Glover and Ball 2013). This paper aims to integrate similar objectives into the optimization of scheduling interventions. However, the problem of scheduling interventions exhibits several particularities. First, unlike ATFM, no standard of equity has been accepted in the industry with respect to scheduling interventions. Second, scheduling interventions may result in flights being rescheduled earlier or later than the times requested by the airlines. This contrasts with the situation in ATFM where flights cannot be moved earlier than their scheduled times, so the schemes of ration-by-schedule and schedule compression do not have any direct analogs in the context of scheduling interventions. It is thus necessary to propose new metrics of interairline equity and to develop new modeling frameworks for scheduling interventions.

This topic has motivated ongoing research at slot-controlled airports (Jing and Zografos 2016). In this paper, we provide a novel formulation of the equity problem in scheduling interventions, specifically aimed at balancing the average per-flight displacement of flights from their requested times fairly among the airlines. This formulation is based on lexicographic utility maximization that extends the min–max equity formulation from Bertsimas, Farias, and Trichakis (2011, 2012) and satisfies some desirable theoretical properties (see details in Section 3.1). We also present the first theoretical results on the trade-off (or the lack thereof) between efficiency and equity in scheduling interventions, and apply this modeling framework to a case study at a major U.S. airport.

### 1.3. Contributions

The main contribution of this paper consists of developing and solving a set of optimization models that incorporate interairline equity considerations in airport scheduling interventions. Our approach builds on the *Integrated Capacity Utilization and Scheduling Model* (*ICUSM*) from Jacquillat and Odoni (2015a) that optimizes such interventions through temporal shifts in demand, but extends it in a way that balances scheduling adjustments equitably among the airlines. We name the resulting model the Integrated Capacity Utilization and Scheduling Model with Equity Considerations (ICUSM-E).

Specifically, this paper makes the following contributions:

- *Quantifying and optimizing the trade space between performance attributes for scheduling interventions*. We identify on-time performance (i.e., mitigating airport congestion), efficiency (i.e., meeting airline scheduling preferences), and equity (i.e., balancing scheduling adjustments fairly among the airlines) as three performance attributes. We develop quantitative indicators for each of them, using a unified framework of scheduling interventions. We then formulate a tractable multilevel architecture to characterize and optimize the trade space between on-time performance, efficiency, and equity in airport scheduling interventions.
- *Characterizing conditions under which efficiency and equity can be jointly maximized*. We show that, in the absence of network connections and in the case where all flights are equally costly (or equally inconvenient) to reschedule, efficiency and equity can be jointly maximized when the interventions involve only reductions in demand, or when the interventions involve only temporal shifts in demand and one of the following conditions is satisfied: (i) any three-period interval has enough capacity to accommodate demand over the same interval, or (ii) the schedules of flights of the different airlines exhibit the same intraday patterns. We then describe instances where the differentiated schedules of flights, network connections, or unequal flight valuations can give rise to a trade-off between efficiency and equity.
- *Generating and solving real-world full-scale computational scenarios at John F. Kennedy Airport* (*JFK*). We show that, under a wide range of realistic and hypothetical

scheduling conditions, the consideration of efficiency-based objectives exclusively in airport scheduling interventions may lead to highly inequitable outcomes, but that interairline equity can be achieved at no (or minimal) efficiency losses. This suggests that existing approaches for scheduling interventions can be extended to include interairline equity considerations.

### 1.4. Outline

The remainder of this paper is organized as follows. In Section 2, we summarize the Integrated Capacity Utilization and Scheduling Model (ICUSM) from Jacquillat and Odoni (2015a), and discuss its limitations related to interairline equity. In Section 3, we formulate the Integrated Capacity Utilization and Scheduling Model with Equity Considerations (ICUSM-E) that incorporates interairline equity objectives. Section 4 provides a theoretical discussion of the trade-off, or lack of it, between efficiency and equity in the context of airport scheduling interventions. In Section 5, we show computational results from a case study at JFK Airport. Section 6 concludes.

## 2. Base Model of Scheduling Interventions

We first summarize the ICUSM from Jacquillat and Odoni (2015a), which provides the baseline for the optimization of our scheduling interventions. Moreover, this section introduces notations that will be used throughout this paper.

### 2.1. Formulation

The ICUSM considers a two-step process, under which the airlines provide a schedule of flights to a central decision maker, who may then propose scheduling adjustments to reduce above-capacity scheduling at an airport, and hence reduce anticipated delays. We denote by $\Pi$ the airport where the scheduling interventions are considered. No flight is eliminated, and delays are reduced by distributing flights more evenly over the day. The model takes as inputs each airline's preferred schedule of flights (e.g., the schedule in the absence of demand management) and estimates of the capacity of airport $\Pi$ (i.e., the expected number of movements that can be operated per unit of time in various operating conditions). It determines which flights to reschedule to later or earlier times to minimize the displacement from the airlines' preferred schedule of flights, subject to scheduling and network connectivity constraints and on-time performance constraints. Scheduling and network connectivity constraints ensure that the airlines' flight networks are minimally affected, and on-time performance constraints ensure that expected arrival and departure queue lengths do not exceed prespecified targets. The modeling framework of the ICUSM integrates into an integer-programming model of scheduling interventions, a stochastic queuing model of airport congestion, and a dynamic programming model of airport capacity utilization.

#### 2.1.1. Inputs.

$\mathcal{T}$ = set of 15-minute time periods, indexed by $t = 1, \dots, T$;

$\mathcal{F}$ = set of flights, indexed by $i = 1, \dots, F$;

$\mathcal{F}^{\text{arr}}/\mathcal{F}^{\text{dep}}$ = set of flights $i \in \mathcal{F}$ scheduled to land/take off at airport $\Pi$;

$\mathcal{C} \subset \mathcal{F} \times \mathcal{F}$ = set of ordered flight pairs $(i, j) \in \mathcal{F} \times \mathcal{F}$ such that there is a connection from $i$ to $j$;

$$\mathcal{S}_{it}^{\text{arr}}/\mathcal{S}_{it}^{\text{dep}} = \begin{cases} 1 & \text{if flight } i \text{ is scheduled to land/take off no earlier than period } t; \\ 0 & \text{otherwise;} \end{cases}$$

$t_{ij}^{\min}/t_{ij}^{\max}$ = minimum/maximum connection time between flight $i$ and flight $j$ $\forall (i, j) \in \mathcal{C}$.

A connection refers to any pair of flights between which a minimum and/or a maximum time must be maintained to enable an aircraft, passengers, or a crew to connect. Note that the set of flights considered in the model may include flights that are not scheduled to land or take off at the airport $\Pi$ where the scheduling interventions are applied, that is, $\mathcal{F}^{\text{arr}} \cup \mathcal{F}^{\text{dep}}$ may be a strict subset of $\mathcal{F}$. This arises from the need to maintain feasible connections in a network of airports.

#### 2.1.2. Variables.

$$w_{it}^{\text{arr}}/w_{it}^{\text{dep}} = \begin{cases} 1 & \text{if flight } i \text{ is rescheduled to land/take off no earlier than period } t; \\ 0 & \text{otherwise;} \end{cases}$$

$u_i$ = displacement (positive or negative) of flight $i$, as number of 15-minute periods;

$\lambda_t^{\text{arr}}/\lambda_t^{\text{dep}}$ = number of arrivals/departures scheduled at airport $\Pi$ during period $t$, after rescheduling.

By convention, we assume that $w_{i,T+1}^{\text{arr}} = w_{i,T+1}^{\text{dep}} = 0$ for all flights $i \in \mathcal{F}$.

**2.1.3. Objective.** The model minimizes, first, the *largest* schedule displacement that any flight will sustain, denoted by $\delta$ (Equation (1)), and, second, the *total* schedule displacement, denoted by $\Delta_0$ (Equation (2))

$$\delta = \max_{i \in \mathcal{F}} |u_i|, \tag{1}$$

$$\Delta_0 = \sum_{i \in \mathcal{F}} |u_i|. \tag{2}$$

**2.1.4. Constraints.** For notational ease, a parameter $\kappa$ refers to either the "arr" or the "dep" superscript of the inputs and variables defined earlier

$$w_{it}^{\kappa} \geq w_{i,t+1}^{\kappa}, \quad \forall\, i \in \mathcal{F}, \forall\, \kappa \in \{\text{arr}, \text{dep}\}, \forall\, t \in \mathcal{T}, \tag{3}$$

$$w_{i1}^{\kappa} = 1, \quad \forall\, i \in \mathcal{F}, \forall\, \kappa \in \{\text{arr}, \text{dep}\}, \tag{4}$$

$$\sum_{t \in \mathcal{T}} (w_{it}^{\kappa} - S_{it}^{\kappa}) = u_i, \quad \forall\, i \in \mathcal{F}, \forall\, \kappa \in \{\text{arr}, \text{dep}\}, \tag{5}$$

$$\sum_{t \in \mathcal{T}} (w_{jt}^{\mathrm{dep}} - w_{it}^{\mathrm{arr}}) \geq t_{ij}^{\min}, \quad \forall (i, j) \in \mathscr{C}, \tag{6}$$

$$\sum_{t \in \mathcal{T}} (w_{jt}^{\mathrm{dep}} - w_{it}^{\mathrm{arr}}) \leq t_{ij}^{\max}, \quad \forall (i, j) \in \mathscr{C}, \tag{7}$$

$$\sum_{i \in \mathcal{F}^k} (w_{it}^{\kappa} - w_{i,t+1}^{\kappa}) = \lambda_t^{\kappa}, \quad \forall t \in \mathcal{T}, \forall \kappa \in \{\mathrm{arr}, \mathrm{dep}\}. \tag{8}$$

Constraint (3) ensures that $w^{\mathrm{arr}}$ and $w^{\mathrm{dep}}$ are nonincreasing in $t$. Constraint (4) ensures that no flight is eliminated. Constraint (5) defines flight displacement as the difference between rescheduled and original scheduled times, and ensures that the scheduled block times are left unchanged. Constraints (6) and (7) maintain connection times within the specified ranges. Constraint (8) defines the aggregate schedule of flights (i.e., the number of scheduled arrivals and departures per time period). Next, we summarize the model of airport congestion that quantifies arrival and departure queue lengths as a function of the schedule of flights (i.e., of $\lambda_t^{\mathrm{arr}}$ and $\lambda_t^{\mathrm{dep}}$).

Arrival and departure queues are quantified by means of stochastic $M(t)/E_3(t)/1$ queuing systems, that is, the demand processes are modeled as time-varying Poisson processes, and the service processes are modeled as time-varying Erlang processes of order 3. For each period $t$, the arrival and departure demand rates are determined by flight schedules, that is, they are equal to $\lambda_t^{\mathrm{arr}}$ and $\lambda_t^{\mathrm{dep}}$, respectively. Service rates are constrained by airport capacity. To capture the endogeneity of the service rates with respect to airport capacity and flight schedules, a control of capacity utilization procedures is integrated into the stochastic queuing model of congestion. It is formulated as a finite-horizon dynamic programming model that minimizes congestion costs for a given schedule of flights (Jacquillat, Odoni, and Webster 2017). At the beginning of each 15-minute period, the control selects the runway configuration and the balance of arrival and departure service rates for that period, under capacity constraints, as a function of observed arrival and departure queue lengths, the runway configuration in use, and wind and weather conditions. The combination of the stochastic queuing model and the dynamic programming model of capacity utilization provides an integrated model of airport congestion that is computationally efficient and that approximates the magnitude and dynamics of delays at busy U.S. airports with good accuracy (Jacquillat and Odoni 2015b). This quantifies, in turn, the relationship (denoted by $q$) between flight schedules and flight delays, represented as follows (where $A_t$ and $D_t$ denote the random variables that represent the arrival and departure queue lengths at the end of period $t$):

$$q\colon (\lambda_1^{\mathrm{arr}}, \ldots, \lambda_T^{\mathrm{arr}}, \lambda_1^{\mathrm{dep}}, \ldots, \lambda_T^{\mathrm{dep}}) \mapsto (A_1, \ldots, A_T, D_1, \ldots, D_T). \tag{9}$$

Based on this relationship, the ICUSM aims to ensure that, at any time of the day, the expected arrival and departure queue lengths do not exceed the prespecified limits, denoted by $A_{\mathrm{MAX}}$ and $D_{\mathrm{MAX}}$, respectively (Constraints (10) and (11) below)

$$E(A_t) \leq A_{\mathrm{MAX}}, \quad \forall t \in \mathcal{T}, \tag{10}$$

$$E(D_t) \leq D_{\mathrm{MAX}}, \quad \forall t \in \mathcal{T}. \tag{11}$$

However, these constraints cannot be directly formulated into the integer-programming scheduling model described above, as the queuing dynamics (Equation (9)) depend nonlinearly on the schedule of flights, hence on the model's decision variables. The solution of the ICUSM therefore relies on an algorithm that iterates between the integer-programming model of scheduling interventions, the dynamic programming model of capacity utilization, and the stochastic queuing model of airport congestion outlined above, until it converges to the optimal value of the schedule displacement. For any given expected queue length targets, this algorithm terminates in 10–15 iterations and in 90 minutes to several hours, depending on model parameters. For more details, we refer the reader to Jacquillat and Odoni (2015a).

### 2.2. Interairline Equity Concerns

The ICUSM provides a modeling framework for optimizing congestion-mitigating scheduling interventions. Its implementation quantifies the trade-off between schedule displacement (i.e., $\delta$ and $\Delta_0$) and peak expected queue length limits (i.e., $A_{\mathrm{MAX}}$ and $D_{\mathrm{MAX}}$). However, it does not account for interairline equity considerations. Its two-stage formulation (characterized by objectives (1) and (2)) ensures equity at the flight level, that is, no flight is disproportionately displaced. However, it does *not* ensure equity at the airline level. In turn, its solution may penalize one airline (or a small subset of airlines) disproportionately.

To illustrate this, Table 1 shows an example with 12 flights scheduled by two airlines over a one-hour interval, with twice as many flights scheduled by Airline 1 (eight flights) as by Airline 2 (four flights). We consider simple on-time performance constraints that impose that no more than three flights (arrivals and departures) can be scheduled during any 15-minute period. The original schedule indicates the preferred schedule of each flight, as requested by the airlines. A total of six flights (three flights between 8:00 and 8:14, and three flights between 8:30 and 8:44) need to be rescheduled to comply with the limit of three flights per period. Schedule 1 provides a solution where six flights from Airline 1 are rescheduled. This solution is clearly inequitable, as it assigns all of the displacement to one airline, and leaves the schedule of the other airline unchanged. By contrast, Schedule 2 provides a

**Table 1.** Example of Inequitable and Equitable Scheduling Interventions

| Movement | Airline | Original schedule | Schedule 1 | Schedule 2 | Schedule 3 |
|---|---|---|---|---|---|
| Arrival | Airline 1 | 8:00 | <u>8:15</u> | <u>8:15</u> | <u>8:15</u> |
| Arrival | Airline 2 | 8:00 | 8:00 | 8:00 | 8:00 |
| Departure | Airline 1 | 8:05 | <u>8:20</u> | 8:05 | 8:05 |
| Departure | Airline 2 | 8:05 | 8:05 | <u>8:20</u> | <u>8:20</u> |
| Departure | Airline 1 | 8:10 | <u>8:25</u> | <u>8:25</u> | <u>8:25</u> |
| Arrival | Airline 1 | 8:10 | 8:10 | 8:10 | 8:10 |
| — | — | — | — | — | — |
| Arrival | Airline 1 | 8:30 | <u>8:45</u> | 8:30 | <u>8:45</u> |
| Arrival | Airline 2 | 8:30 | 8:30 | <u>8:45</u> | 8:30 |
| Departure | Airline 1 | 8:35 | <u>8:50</u> | <u>8:50</u> | <u>8:50</u> |
| Arrival | Airline 1 | 8:35 | 8:35 | 8:35 | 8:35 |
| Departure | Airline 1 | 8:40 | <u>8:55</u> | 8:40 | 8:40 |
| Departure | Airline 2 | 8:40 | 8:40 | <u>8:55</u> | <u>8:55</u> |
| — | — | — | — | — | — |

solution where three flights from Airline 1 and three flights from Airline 2 are rescheduled. This solution does not appear equitable either, as it assigns a similar displacement to the two airlines, even though Airline 1 has more flights scheduled at the airport than Airline 2. It thus imposes a greater per-flight displacement to the schedule of Airline 2 than to that of Airline 1. Schedule 3, then, provides an equitable solution that displaces exactly twice as many flights from Airline 1 (four flights) as from Airline 2 (two flights). The modeling architecture presented in Section 3 formalizes the measurement of interairline equity and integrates it into the optimization model for airport scheduling interventions.

## 3. Multicriteria Modeling Architecture

We now present our Integrated Capacity Utilization and Scheduling Model with Equity Considerations (ICUSM-E). The model structure, the decision variables, and the scheduling and network connectivity constraints are identical to those in the ICUSM, but the main difference lies in the objectives of scheduling interventions. In addition to the notations introduced in Section 2, we partition the set of flights scheduled at airport $\Pi$, that is, $\mathcal{F}^{\text{arr}} \cup \mathcal{F}^{\text{dep}}$, into subsets scheduled by the different airlines

$\mathcal{A}$ = set of airlines, indexed by $\{1, \ldots, A\}$;
$\mathcal{F}_a$ = set of flights scheduled by airline $a$ at airport $\Pi$.

With these notations, we have $\mathcal{F}_{a_1} \cap \mathcal{F}_{a_2} = \varnothing$ for all $a_1, a_2 \in \mathcal{A}$ such that $a_1 \neq a_2$, and $\bigcup_{a\in\mathcal{A}} \mathcal{F}_a = \mathcal{F}^{\text{arr}} \cup \mathcal{F}^{\text{dep}}$.

We also introduce parameters $v_i$ for each flight $i \in \mathcal{F}$ to characterize "flight valuations," reflecting airlines' preferences regarding which flights to reschedule. Flights with lower valuations can be thought of as less "costly" to reschedule, or as the flights that exhibit more timetabling flexibility. Note that the current setting where "a flight is a flight" is a special case, where $v_i = 1$ for all $i \in \mathcal{F}$. Even though the valuations $v_i$ are not available to the central decision makers in current mechanisms for airport scheduling interventions, they could be considered in future extensions of these mechanisms. For instance, they could be the result of nonmonetary processes that would allow the airlines to indicate their preferences through ranking or credit allocation. Alternatively, they could result from an auction-based mechanism where airlines would submit a bid for each flight $i$, and pay an access fee that is discounted by a fixed percentage for each period of displacement (e.g., if an airline bids $x$ for an 8:00 flight, it would pay an access fee equal to $x$ if it is scheduled at 8:00, $(1-\alpha)x$ if it is scheduled at 7:45 or 8:15, $(1-2\alpha)x$ if it is scheduled at 7:30 or 8:30, etc., where $\alpha < 1$). This is similar to the mechanisms proposed in energy (Newbery 2003; Stern and Turvey 2003), railway transportation (Pena-Alcaraz 2015), or telecommunications markets (Hoffman 2010). While the design of such mechanisms is beyond the scope of this paper, our modeling approach incorporates interairline equity objectives in instances with either identical or differentiated flight valuations

$$v_i = \text{valuation of flight } i, \quad \forall\, i \in \mathcal{F}.$$

As mentioned in Section 1, this paper assumes that these inputs truthfully reflect airlines' scheduling preferences. This is motivated by the fact that the limited scheduling adjustments (timetabling changes of 15 or 30 minutes) considered here, provide less obvious incentives for untruthful behaviors than, for instance, mechanisms that involve the rejection of some flight scheduling requests (as in the latter case, airlines might be incentivized to request more peak-hour flights than what they would truly like to schedule, due to the anticipation that some will be rejected), or more significant flight displacements. Nonetheless, even in cases with 15- or 30-minute timetabling changes, there may still exist opportunities for the airlines to gain strategic advantages, for instance, by concentrating their scheduling requests at or close to peak hours. The identification of such gaming behaviors, and their mitigation, represent important avenues for future research. Similarly, the effect of the interairline equity considerations introduced in this paper on airlines' flight scheduling requests (e.g., whether they increase or decrease the gaming opportunities; whether they lead to more or less peaked schedules, to higher or lower schedule frequencies, etc.) is beyond the scope of this paper and can be addressed in future work.

### 3.1. Performance Attributes

We consider the following three performance attributes of scheduling interventions: on-time performance, efficiency, and interairline equity. On-time performance and efficiency extend the notions of expected queue length limits and schedule displacement, respectively, that are considered in the ICUSM, while the notion of equity is added to this framework for the first time in this paper.

The consideration of these three performance attributes makes the optimization of scheduling interventions a multiobjective optimization problem. First, each attribute comprises several dimensions (see details below). Moreover, there exists a trade-off between efficiency and on-time performance, quantified by the ICUSM: the larger the schedule displacement, the larger the potential delay reductions (up to a limit). Finally, there may be, for given on-time performance objectives, a trade-off between efficiency and equity.

**3.1.1. On-Time Performance.** This refers to the ability to mitigate airport congestion. We quantify on-time performance by a nondecreasing function of the arrival and departure queue lengths $A_t$ and $D_t$ (which depend on the schedule of flights according to (9)), denoted by $g(A_1, \ldots, A_T, D_1, \ldots, D_T)$. Maximizing on-time performance involves minimizing the function $g$

$$\min\{g(A_1, \ldots, A_T, D_1, \ldots, D_T)\}. \tag{12}$$

In this paper, we quantify on-time performance by the peak expected arrival and departure queue lengths, that is, when $g(A_1, \ldots, A_T, D_1, \ldots, D_T) = (\max_{t\in\mathcal{T}} E(A_t), \max_{t\in\mathcal{T}} E(D_t))$. It is motivated by the objective of controlling the largest delays experienced over the day. Other possible functions that could have been considered include minimizing the total delay experienced over the full day, the 95th percentile of the peak queue lengths, etc. Our choice of the peak expected queue lengths is motivated by the prior literature on airport scheduling interventions (Jacquillat and Odoni 2015a).

**3.1.2. Efficiency.** This refers to the ability to meet airline scheduling preferences. Since no flight is eliminated, efficiency is measured by the displacement from the schedule of flights requested by the airlines. We consider two efficiency objectives. First, we define min–max *efficiency* as the largest displacement sustained by any flight. As in Section 2, we denote it by $\delta$. Second, we define *weighted efficiency* as the weighted sum of schedule displacements sustained by all flights, and we denote it by $\Delta$. Weighted efficiency generalizes the total displacement $\Delta_0$ considered in the ICUSM in a way that accounts for flight valuations. Directionally, maximizing efficiency involves minimizing $\delta$ and/or $\Delta$

$$\delta = \max_{i\in\mathcal{F}} |u_i| \implies \min \delta, \tag{13}$$

$$\Delta = \sum_{i\in\mathcal{F}} v_i |u_i| \implies \min \Delta. \tag{14}$$

**3.1.3. Interairline Equity.** This refers to the ability to balance schedule displacement fairly among the airlines. We describe each airline's disutility as the weighted average of per-flight displacements, denoted by $\sigma_a$. Ideally, the weighted average of per-flight displacements would be the same for all airlines, that is, the weighted sum of displacements borne by any airline would be proportional to its number of flights scheduled at airport $\Pi$. To maximize interairline equity, we minimize airline disutilities lexicographically, that is, we first minimize the largest airline disutility, then the second largest, etc.

$$\sigma_a = \frac{1}{|\mathcal{F}_a|} \sum_{i\in\mathcal{F}_a} v_i |u_i|, \ \ \forall\, a \in \mathcal{A} \implies \operatorname{lex\,min} \sigma. \tag{15}$$

We denote the largest airline disutility by $\Phi$

$$\Phi = \max_{a\in\mathcal{A}} \sigma_a. \tag{16}$$

This lexicographic approach to interairline equity maximization extends the min–max formulation proposed by Bertsimas, Farias, and Trichakis (2011, 2012), which maximizes the largest utility, that is, minimizes the largest of the weighted average per-flight displacements borne by any airline here. This choice is motivated by three main factors. From a theoretical standpoint, it extends the solution of Kalai and Smorodinsky (1975) for the two-player bargaining problem, which is the only solution that satisfies the axioms of Pareto optimality (i.e., no other solution can improve the utility of one airline, without reducing that of another airline), symmetry (i.e., the "order" of the airline does not affect the results), affine invariance (i.e., it does not depend on the choice of equivalent utility representations), and monotonicity (i.e., if the total displacement is reduced, then the utility of any of the airlines should not decrease). From a computational standpoint, this approach can be formulated using linear mixed-integer programming models, which ensures far greater computational efficiency than alternative nonlinear approaches. From a practical standpoint, it has a broad application in multicriteria decision-making problems, for instance in the context of resource allocation (Klein, Luss, and Smoth 1992; Luss 1999), telecommunications (Ogryczak, Pioro, and Tomaszewski 2005), and electricity (Sun 2011).

Note that this is one among several other possible approaches to equity quantification and optimization. For instance, Bertsimas, Farias, and Trichakis (2012) also propose a broader class of welfare functions $W_\alpha(\sigma) = \sum_{a\in\mathcal{A}} \sigma_a^{1-\alpha}/(1-\alpha)$, for each $\alpha \geq 0$, $\alpha \neq 1$, and $W_1(\sigma) = \sum_{a\in\mathcal{A}} \log(\sigma_a)$. This establishes a continuum between the efficiency-maximizing outcome ($\alpha = 0$) and the min–max equity scheme that we extend in this paper ($\alpha \to \infty$). Other formulations could also be based on the *dispersion* of agent utilities (e.g., by minimizing functions like $\sum_{a,a' \in \mathscr{A}} |\sigma_a - \sigma_{a'}|$, $\max_{a,a' \in \mathscr{A}} |\sigma_a - \sigma_{a'}|$ or $\sum_{a \in \mathscr{A}} (\sigma_a - \bar{\sigma})^2$, where $\bar{\sigma}$ denotes the average disutility across all airlines; Leclerc, McLay, and Mayorga 2012). Unlike our lexicographic approach, these metrics explicitly minimize the variations in airline disutilities, and aim to achieve an outcome where $\sigma_a = \sigma_{a'}$ for all $a, a' \in \mathscr{A}$. However, these dispersion-based metrics do *not* satisfy the condition of monotonicity. For instance, in a two-airline environment, they (i) are indifferent between a solution that assigns a disutility of 0.10 to both airlines, and another solution that assigns a disutility of 0.09 to both airlines; and (ii) would favor a solution that assigns a disutility of 0.10 to both airlines over another solution that assigns a disutility of 0.10 to one airline and 0.09 to the other, or even over another solution that assigns a disutility of 0.095 to one airline and 0.09 to the other. The objective of finding an equity scheme that satisfies the condition of Pareto optimality, affine invariance, symmetry, and monotonicity motivates our lexicographic approach to equity maximization.

Moreover, our approach to interairline equity optimization is focused on balancing the schedule displacement fairly among the airlines, and on ensuring that no systematic bias in favor of or against one airline (or a group of airlines) is introduced in the scheduling interventions process. Note, however, that interairline equity may include other considerations. For example, existing processes at slot-controlled airports outside the United States have provisions for grandfathered rights, slots for new entrants, and series of slots, to incorporate certain notions of equity. These considerations are incorporated to prioritize flights with historical precedence, enable airport access for new entrant airlines, and promote schedule regularity, respectively. The integration of such other considerations in a broader framework for interairline equity represents an important avenue for future research.

### 3.2. Multilevel Modeling Approach

We characterize the trade space between on-time performance, efficiency, and equity in airport scheduling interventions. To provide a transparent and optimal characterization of this trade space, we aim to find its Pareto frontier, that is, the set of solutions such that no other feasible solution could improve at least one of the three objectives without worsening the others. This representation of the trade space is flexible enough to be used by system managers and policy makers to select the most appropriate level of compromise between these objectives. To this end, we develop a multilevel optimization approach that (i) fixes on-time performance targets; (ii) maximizes min–max efficiency under on-time performance targets; and (iii) produces the Pareto frontier of equity and weighted efficiency under on-time performance targets and min–max efficiency targets. This multilevel structure is also consistent with industry practice. For instance, air traffic flow management typically aims, first, to maximize system safety, then to minimize total delays in the system, then to equitably distribute the delay costs across various airlines.

First, we fix on-time performance targets. We denote by $A_{\text{MAX}}$ and $D_{\text{MAX}}$ the limits on the peak expected arrival and departure queue lengths, respectively. The corresponding on-time performance constraints are identical to those in the ICUSM (Constraints (10) and (11)). We then aim to find, as described next, the "best" schedule (in terms of efficiency and equity) that meets these constraints.

Second, we determine the schedule of flights that maximizes efficiency, subject to scheduling, network connectivity, and on-time performance constraints. To avoid large flight displacements, we formulate the efficiency-maximizing problem by, first, maximizing min–max efficiency (i.e., minimizing $\delta$), and, second, maximizing weighted efficiency (i.e., minimizing $\Delta$), under optimal min–max efficiency. This is consistent with the existing literature (Pyrgiotis and Odoni 2016; Jacquillat and Odoni 2015a), and the subject of ongoing research (Zografos, Madas, and Androutsopoulos 2017; Ribeiro et al. 2018). This is expressed in Problems P1 and P2:

**P1.** We minimize min–max efficiency metric $\delta$, subject to scheduling, network connectivity, and on-time performance constraints. We denote by $\delta^*$ its optimal value

min $\delta$ (Equation (13))
s.t. Scheduling and network connectivity constraints: (3) to (8),
On-time performance constraints: (9) to (11).

**P2.** We minimize weighted efficiency metric $\Delta$, subject to scheduling, network connectivity, and on-time performance constraints, and subject to the constraint that no flight may be displaced by more than $\delta^*$ (i.e., ensuring optimal min–max efficiency). We denote by $\Delta^*$ its optimal value

min $\Delta$ (Equation (14))
s.t. Scheduling and network connectivity constraints: (3) to (8),
On-time performance constraints: (9) to (11),
Min–max efficiency objectives: $|u_i| \le \delta^*, \forall\, i \in \mathscr{F}$.

Third, we maximize interairline equity, subject to scheduling constraints, network connectivity constraints, on-time performance constraints, and efficiency targets. This is formulated in the class of Problems P3($\rho$) described below:

**P3($\rho$).** We fix weighted efficiency targets, and we lexicographically minimize airline disutilities, subject to scheduling, network connectivity, on-time performance, min–max efficiency, and weighted efficiency constraints. We characterize the Pareto-optimal trade space between weighted efficiency and equity by varying the weighted efficiency target. Specifically, we impose that min–max efficiency must be optimal (i.e., no flight may be rescheduled by more than $\delta^*$) and we denote by $\rho \in [0, \infty)$ the relative loss in weighted efficiency, compared to $\Delta^*$, that is allowed (i.e., the weighted displacement must not exceed $(1+\rho)\Delta^*$). When $\rho \to \infty$, we only maximize equity (without any weighted efficiency consideration). When $\rho = 0$, we maximize equity, under optimal min–max efficiency and optimal weighted efficiency. For any given value of $\rho$, we denote by $\sigma^*(\rho)$ the optimal vector of airline disutilities and by $\Phi^*(\rho) = \max_{a\in\mathcal{A}} \sigma^*_a(\rho)$ the largest airline disutility

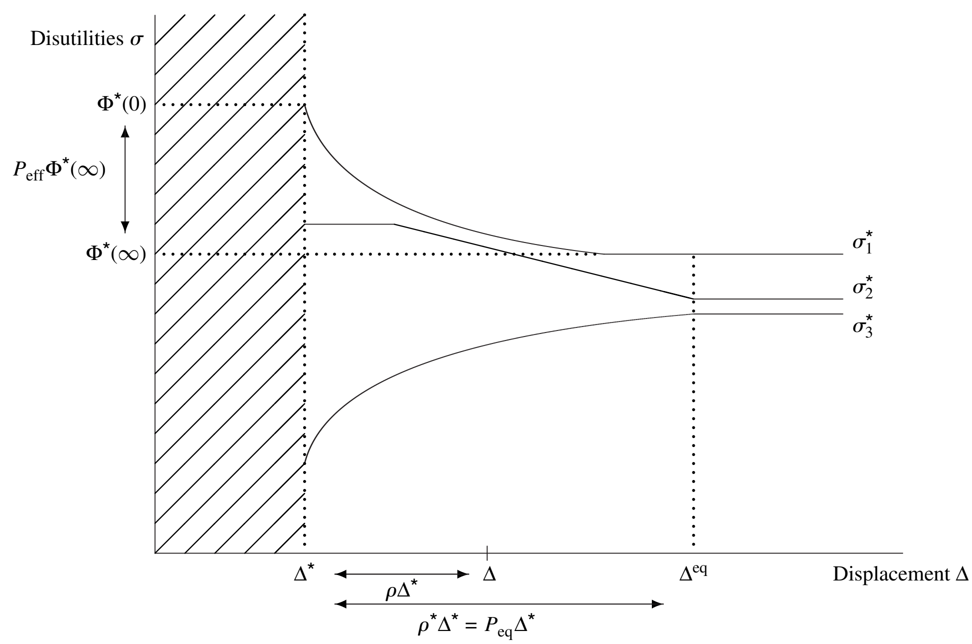
lex min $\sigma$ (Equation (15))
s.t. Scheduling and network connectivity constraints: (3) to (8),
On-time performance constraints: (9) to (11),
Min–max efficiency objectives:
$|u_i| \le \delta^*, \forall\, i \in \mathcal{F}$,
Weighted efficiency objectives:
$$\sum_{i\in\mathcal{F}} v_i |u_i| \le (1+\rho)\Delta^*.$$

Problems P1, P2, and P3($\rho$) together determine the Pareto frontier of the trade space between efficiency, equity, and on-time performance. First, variations in the on-time performance targets $A_{\text{MAX}}$ and $D_{\text{MAX}}$ quantify the trade-off between the costs of scheduling interventions (in terms of inefficiency and inequity) and delay reductions. Second, for any on-time performance targets $A_{\text{MAX}}$ and $D_{\text{MAX}}$, varying the parameter $\rho$ quantifies the trade-off (or lack thereof) between weighted efficiency and interairline equity (under optimal min–max efficiency).

We denote by $\rho^*$ the minimum relative loss in weighted efficiency required to attain optimal equity, that is, $\rho^* = \min\{\rho \mid s(\sigma^*(\rho)) = s(\sigma^*(\infty))\}$ (where $s(\sigma)$ denotes the sorted vector of airline disutilities $\sigma$), and by $\Delta^{\text{eq}}$ the smallest equity-maximizing value of $\Delta$, that is, $\Delta^{\text{eq}} = (1+\rho^*)\Delta^*$. With these notations, the "price of equity" is characterized by $P_{\text{eq}} = (\Delta^{\text{eq}} - \Delta^*)/\Delta^* = \rho^*$. Similarly, the "price of efficiency" is defined by $P_{\text{eff}} = (\Phi^*(0) - \Phi^*(\infty))/\Phi^*(\infty)$.

Figure 1 illustrates our approach to maximizing weighted efficiency and interairline equity, for given on-time performance targets $A_{\text{MAX}}$ and $D_{\text{MAX}}$, and the optimal value of min–max efficiency $\delta^*$. Specifically, it shows hypothetical variations in three airlines' disutilities ($\sigma_1$, $\sigma_2$, and $\sigma_3$) as a function of the weighted efficiency target $\Delta = (1+\rho)\Delta^*$. By construction, the region on the left side of $\Delta^*$ is infeasible, that is, the weighted schedule displacement must be at least $\Delta^*$. Moreover, the largest airline disutility $\Phi$ is a nonincreasing function of the value of weighted efficiency $\Delta$ (i.e., of $\rho$). Note that the other airlines' utilities (here, $\sigma_2$ and $\sigma_3$) may increase, decrease, or stay constant, as $\Phi$ is reduced. When the largest airline disutility $\Phi$ attains its optimal value, the second-largest disutility may still be larger than its optimal value. In this case, further increases in $\rho$ may yield further improvements in the lexicographic minimization of airline disutilities. Optimal equity is attained when the largest, second largest,

**Figure 1.** A Schematic Trade Space Between Weighted Efficiency and Equity



third largest, etc., disutilities have all reached their optimal values (i.e., the values that would be obtained without any efficiency consideration, or with $\rho \to \infty$). This representation shows the price of efficiency and the price of equity as the relative difference between $\Phi^*(\infty)$ and $\Phi(0)$, and between $\Delta^*$ and $\Delta^{eq}$, respectively. Note that Figure 1 shows an instance where the order of airline disutilities remains identical for all values of $\rho$ (i.e., in this case, $\sigma_1^*(\rho) > \sigma_2^*(\rho) > \sigma_3^*(\rho)$ for all $\rho \geq 0$), but this need not be the case in general (i.e., the lexicographic minimization of airline disutilities may be such that the curves intersect).

### 3.3. Solution Architecture

As discussed in Section 2, the on-time performance constraints are not linear. Solving Problems P1, P2, and P3($\rho$) thus requires a solution algorithm, adapted from Jacquillat and Odoni (2015a), that iterates between the integer programming models of scheduling interventions (Sections 2.1 and 3.2), the dynamic programming model of capacity utilization (Section 2.1), and the stochastic queuing model of airport congestion (Section 2.1). For solving Problem P1, we search for the optimal maximum flight displacement $\delta^*$ by increasing its value from zero 15-minute periods to one period, then two periods, until a feasible solution is found. For solving Problem P2, we iteratively update an upper bound $\bar{\Delta}$ and a lower bound $\underline{\Delta}$ on the optimal weighted efficiency $\Delta^*$, using binary search. At each iteration, we consider a value of $\Delta = (\bar{\Delta} + \underline{\Delta})/2$, and we update $\bar{\Delta}$ (respectively, $\underline{\Delta}$) to $(\bar{\Delta} + \underline{\Delta})/2$ if the resulting delay estimates meet (respectively, do not meet) the on-time performance constraints. We repeat the process until the following stopping criteria is reached: $(\bar{\Delta} - \underline{\Delta})/\underline{\Delta} \leq \varepsilon_\Delta$. This ensures that the weighted schedule displacement obtained is within $\varepsilon_\Delta$ of the optimal weighted schedule displacement. We use a value of 1% for $\varepsilon_\Delta$.

For solving Problem P3($\rho$), we implement in this paper a similar algorithm. For any given value of $\rho$, we iteratively update an upper bound $\bar{\Phi}$ and a lower bound $\underline{\Phi}$ on the largest airline disutility $\Phi^*(\rho) = \max_{a \in \mathcal{A}} \sigma_a$. At each iteration, we consider a value of $\Phi = (\bar{\Phi} + \underline{\Phi})/2$, and find the schedule that minimizes the maximum queue lengths using a deterministic queuing model, which depends linearly on the schedule of flights, hence, on the model's decision variables. Using the resulting schedule, we compute the expected queue lengths using the dynamic programming model of capacity utilization and the stochastic queuing model of airport congestion. We update $\bar{\Phi}$ (respectively, $\underline{\Phi}$) to $(\bar{\Phi} + \underline{\Phi})/2$ if the resulting delay estimates meet (respectively, do not meet) the on-time performance constraints. We iterate until $(\bar{\Phi} - \underline{\Phi})/\underline{\Phi} \leq \varepsilon_\Phi$, with a value of $\varepsilon_\Phi = 1\%$. We repeat the process to then minimize the second-largest disutility, the third-largest disutility, etc. We provide further details in the online appendix.

Note that, for each airline, this algorithm involves running several iterations between an integer programming model, a dynamic programming model, and a stochastic queuing model. The resulting computational requirements may be significant, especially given that Problem P3($\rho$) needs to be solved repeatedly for several values of $\rho$, for different values of the parameters, and for each day of operations being considered. We thus develop an alternative approach that approximates Problem P3($\rho$) while enhancing computational tractability. Specifically, instead of the on-time performance constraints (Constraints (10) and (11)), we consider scheduling limit constraints (Constraints (17) and (18), defined below). These constraints ensure that, for any period $t$, the number of scheduled arrivals and departures does not exceed the limits denoted by $\widehat{\lambda_t^{arr}}$ and $\widehat{\lambda_t^{dep}}$, respectively. We refer to these constraints as "time-dependent schedule limit constraints"

$$\lambda_t^{arr} \leq \widehat{\lambda_t^{arr}}, \quad \forall\, t \in \mathcal{T}, \tag{17}$$

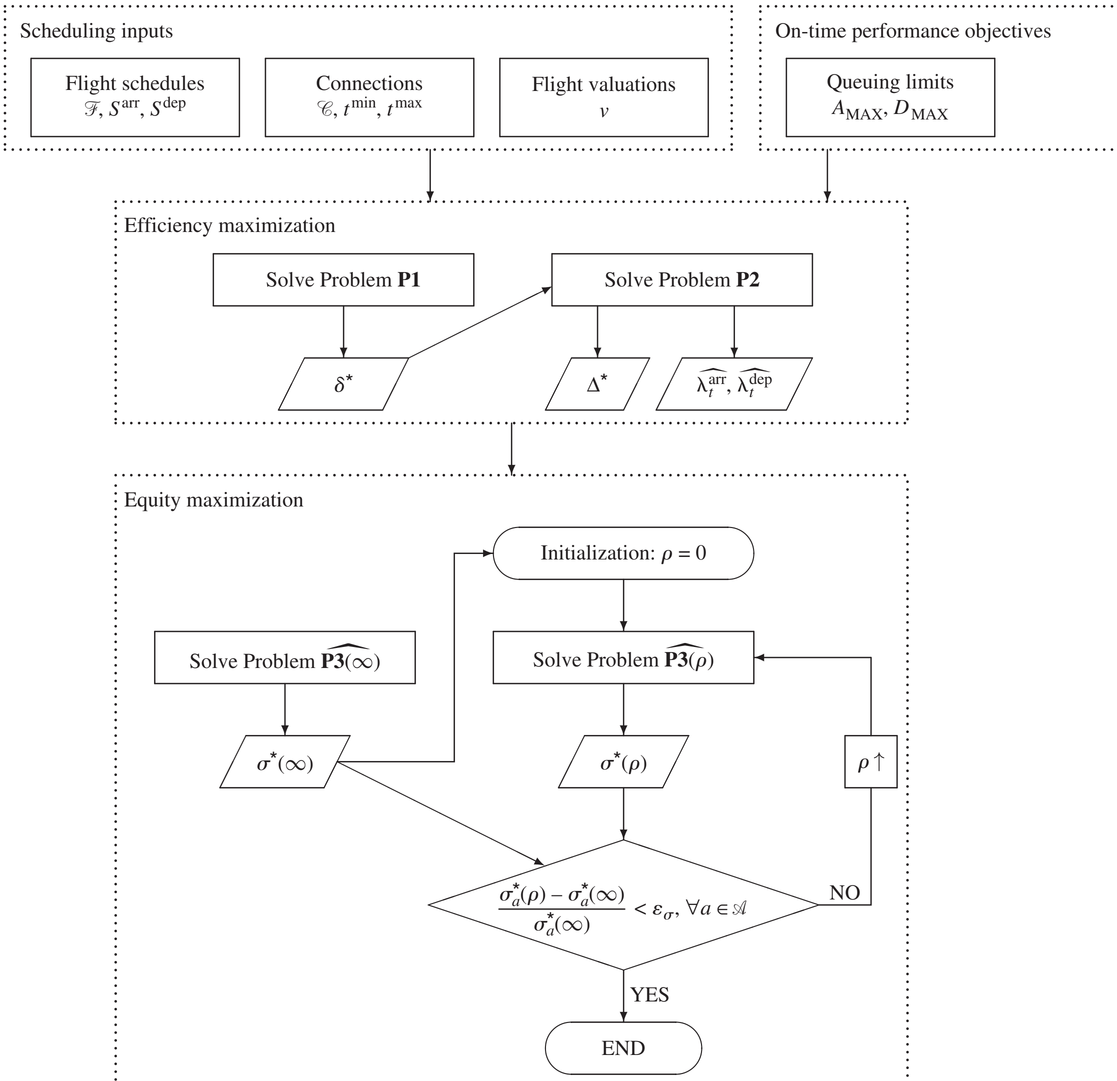
$$\lambda_t^{dep} \leq \widehat{\lambda_t^{dep}}, \quad \forall\, t \in \mathcal{T}. \tag{18}$$

The resulting model is formulated below, and we refer to it as $\widehat{\text{P3}(\rho)}$. Unlike Problem P3($\rho$), Problem $\widehat{\text{P3}(\rho)}$ involves solving a single integer-programming model for each airline

lex min $\sigma$ (Equation (15))
s.t. Scheduling and network connectivity constraints: (3) to (8),
Time-dependent schedule limits constraints: (17) and (18),
Min–max efficiency objectives: $|u_i| \leq \delta^*, \forall\, i \in \mathcal{F}$,
Weighted efficiency objectives: $\sum_{i \in \mathcal{F}} v_i |u_i| \leq (1+\rho)\Delta^*$.

The main challenge lies in setting appropriate values of the scheduling limits $\widehat{\lambda_t^{arr}}$ and $\widehat{\lambda_t^{dep}}$. If set too high, the resulting arrival and departure queue lengths would not meet the on-time performance targets $A_{MAX}$ and $D_{MAX}$, respectively. If set too low, they may not minimize the displacement impact on airline schedules of flights. In this paper, we set the scheduling limits $\widehat{\lambda_t^{arr}}$ and $\widehat{\lambda_t^{dep}}$ equal to the aggregate schedule (i.e., the vector of the number of scheduled arrivals and departures per time period) obtained by solving Problem P2. In other words, we first determine the efficiency-maximizing schedule of flights (by successively minimizing the min–max efficiency and weighted efficiency as described earlier). We then look for flight schedules that achieve the same aggregate schedule (but not necessarily the same schedule for each individual flight), while yielding a Pareto-optimal solution to the trade-off between weighted efficiency and equity.

**Figure 2.** Solution Architecture




By construction, the obtained schedule meets the delay reduction constraints (10) and (11). On the other hand, Constraints (17) and (18) may be more restrictive than Constraints (10) and (11) and may thus yield a suboptimal solution. Nonetheless, the computational results reported in Section 5 show that this approach leads to similar equity levels and much faster computational times, as compared to the more complex approach based on Problem P3($\rho$).

Our full solution architecture is shown in Figure 2. It takes as inputs scheduling data, connections data, and flight valuation data, as well as on-time performance targets $A_{\text{MAX}}$ and $D_{\text{MAX}}$ set by the central decision maker. First, we successively solve Problems P1 and P2, and we store the optimal min–max and weighted efficiency values ($\delta^*$ and $\Delta^*$) and the aggregate schedule ($\widehat{\lambda_t^{\text{arr}}}$ and $\widehat{\lambda_t^{\text{dep}}}$). Second, we solve Problems $\widehat{\text{P3}(\rho)}$ to determine the Pareto frontier of the trade space between weighted efficiency and equity to achieve this aggregate schedule (i.e., $\widehat{\lambda_t^{\text{arr}}}$ and $\widehat{\lambda_t^{\text{dep}}}$). We start by maximizing equity with no weighted efficiency constraint ($\rho \to \infty$). We then maximize equity under optimal weighted efficiency ($\rho = 0$), and we relax progressively the weighted efficiency requirements by increasing $\rho$ in increments of 0.001, until optimal equity is reached. The min–max efficiency is held at its optimal value throughout this process. We use the following stopping criteria: $(\sigma_a^*(\rho) - \sigma_a^*(\infty))/\sigma_a^*(\infty) \le \varepsilon_\sigma$, $\forall a \in \mathcal{A}$, that is, the algorithm terminates when all airlines' disutilities are within $\varepsilon_\sigma$ of their equity-maximizing values. We use a value of 1% for $\varepsilon_\sigma$. This algorithm characterizes, for any pair of on-time performance targets $A_{\text{MAX}}$ and $D_{\text{MAX}}$, and the corresponding optimal min–max efficiency value, (i) the weighted efficiency-maximizing schedule of flights, and (ii) a set of feasible flight schedules that achieve the same aggregate schedule and yield a set of Pareto-optimal solutions to the trade-off between weighted efficiency and equity.

## 4. A Theoretical Discussion on Interairline Equity

We show that, under some conditions on the type of scheduling interventions or on the scheduling inputs provided by the airlines, efficiency and equity can be jointly optimized. We consider in Section 4.1 the case where no network connections need to be maintained and all flights are equally valued. We first show that efficiency and equity can be jointly optimized if the scheduling interventions involve only reductions in demand (i.e., "accept" or "reject" each flight request). Note that this problem is slightly beyond the scope of the model presented in Section 3, but it guarantees that our major theoretical and computational insights apply to a broader set of scheduling interventions than those considered in this paper. We then show that efficiency and equity can also be jointly optimized if the scheduling interventions involve only temporal shifts in demand (i.e., do not eliminate any flights, but may modify their timetabling), if additional scheduling conditions are satisfied. Last, we discuss in Section 4.2 the factors that may violate these conditions, and thus create a trade-off between efficiency and equity in scheduling interventions.

### 4.1. Cases of Joint Maximization of Efficiency and Equity

In this section, we assume that no connections need to be maintained (i.e., $\mathscr{C} = \emptyset$, and therefore the set of relevant flights $\mathscr{F}$ includes only flights scheduled to or from the airport $\Pi$ under consideration) and that all flights are equally valued (i.e., $v_i =$ for all $i \in \mathscr{F}$). In the absence of connections, and with identical flight valuations, all flights in the set $\mathscr{F}$ can be treated the same irrespective of whether they take off or land at the airport $\Pi$ under consideration. So we can simply eliminate the superscripts "arr" and "dep" from the discussion in this section. We denote by $\mathscr{D}_t$ the set of flights scheduled during period $t$ before the scheduling interventions, that is, $\mathscr{D}_t = \{i \in \mathscr{F} \mid S_{it} = 1 \text{ and } S_{i,t+1} = 0\}$. By convention, we assume that $\mathscr{D}_0 = \mathscr{D}_{T+1} = \emptyset$ and $\hat{\lambda}_0 = \hat{\lambda}_{T+1} = 0$. We also denote the positive part of any number $x$ by $x^+ = \max(x, 0)$.

We first consider the case where the scheduling interventions involve only reductions in demand, that is, are restricted to accepting or rejecting individual flight scheduling requests, without changing the timetabling of these flights. We define the corresponding problems of efficiency maximization (EFF-AR) and equity maximization (EQ-AR). The decision variable $y_i$ is equal to 1 if flight $i \in \mathscr{F}$ is rejected, or 0 otherwise. Problem (EFF-AR) minimizes the number of flights rejected and Problem (EQ-AR) lexicographically minimizes the proportion of individual airlines' flights being rejected, subject to the constraint that no more than $\hat{\lambda}_t$ flights may be scheduled during any period $t$

$$\begin{aligned} \min \quad & \sum_{i \in \mathscr{F}} y_i \qquad \text{(EFF-AR)} \\ \text{s.t.} \quad & \sum_{i \in \mathscr{D}_t} (1 - y_i) \le \hat{\lambda}_t, \quad \forall\, t \in \mathscr{T}, \\ & y_i \in \{0, 1\}, \quad \forall\, i \in \mathscr{F}; \end{aligned}$$

$$\begin{aligned} \operatorname{lex\,min} \quad & \left( \frac{1}{|\mathscr{F}_a|} \sum_{i \in \mathscr{F}_a} y_i \right)_{a \in \mathscr{A}} \qquad \text{(EQ-AR)} \\ & \sum_{i \in \mathscr{D}_t} (1 - y_i) \le \hat{\lambda}_t, \quad \forall\, t \in \mathscr{T}, \\ & y_i \in \{0, 1\}, \quad \forall\, i \in \mathscr{F}. \end{aligned}$$

Proposition 1 shows that, in the case where scheduling interventions are based purely on reductions in demand, efficiency and equity can be jointly maximized. The efficient solution is to reject flights that are in excess of the limit, in each of the time periods. The result below shows that this can be done in a way that also maximizes equity.

**Proposition 1.** *There exists a solution that simultaneously solves* (*EFF-AR*) *and* (*EQ-AR*).

**Proof.** The constraint in both (EFF-AR) and (EQ-AR) can be rewritten as $\sum_{i \in \mathscr{D}_t} y_i \ge (|\mathscr{D}_t| - \hat{\lambda}_t)^+$ for all $t \in \mathscr{T}$. Therefore, any solution that involves rejecting exactly $(|\mathscr{D}_t| - \hat{\lambda}_t)^+$ flights in each period $t$ is an optimal solution of (EFF-AR), and the optimal objective function value is $\sum_{t \in \mathscr{T}} (|\mathscr{D}_t| - \hat{\lambda}_t)^+$.

Let us now consider an optimal solution of (EQ-AR). Note that it is well defined because (i) (EQ-AR) clearly admits a feasible solution (e.g., the solution that rejects *all* flights, that is, the solution with $y_i = 1$ for all $i$), and (ii) it admits an optimal solution since (EQ-AR) the problem is discrete and bounded. We assume that $\sum_{i \in \mathscr{F}} y_i > \sum_{t \in \mathscr{T}} (|\mathscr{D}_t| - \hat{\lambda}_t)^+$, that is, $\sum_{t \in \mathscr{T}} (\sum_{i \in \mathscr{D}_t} y_i - (|\mathscr{D}_t| - \hat{\lambda}_t)^+) > 0$. We denote by $\mathscr{J} = \{t \in \mathscr{T} \mid \sum_{i \in \mathscr{D}_t} y_i > (|\mathscr{D}_t| - \hat{\lambda}_t)^+\}$. For each $t \in \mathscr{J}$, we select a subset $\mathscr{K}_t \subseteq \{i \in \mathscr{D}_t \mid y_i = 1\}$ that contains exactly $(\sum_{i \in \mathscr{D}_t} y_i - (|\mathscr{D}_t| - \hat{\lambda}_t)^+)$ elements. We now define a solution $\bar{y}$ as follows:

$$\bar{y}_i = \begin{cases} y_i, & \text{if } i \notin \bigcup_{t \in \mathscr{T}} \mathscr{K}_t, \\ 0, & \text{if } i \in \bigcup_{t \in \mathscr{T}} \mathscr{K}_t. \end{cases}$$

We have, for all periods $t \in \mathscr{T}$, $\sum_{i \in \mathscr{D}_t} \bar{y}_i = (|\mathscr{D}_t| - \hat{\lambda}_t)^+$. Indeed, if $t \notin \mathscr{J}$, then $\sum_{i \in \mathscr{D}_t} \bar{y}_i = \sum_{i \in \mathscr{D}_t} y_i = (|\mathscr{D}_t| - \hat{\lambda}_t)^+$. If $t \in \mathscr{J}$, then $\sum_{i \in \mathscr{D}_t} \bar{y}_i = \sum_{i \in \mathscr{D}_t} y_i - |\mathscr{K}_t| = (|\mathscr{D}_t| - \hat{\lambda}_t)^+$. As a result, the solution $\bar{y}$ is feasible and solves (EFF-AR). Moreover, $\bar{y}_i \le y_i$ for all $i \in \mathscr{F}$, so $\sum_{i \in \mathscr{F}_a} \bar{y}_i \le \sum_{i \in \mathscr{F}_a} y_i$ for all $a \in \mathscr{A}$, and, ultimately, $\bar{y}$ also solves (EQ-AR) to optimality. □

We now turn to the case where the scheduling interventions involve temporal shifts in demand—the case considered in the rest of this paper, which is more consistent with current practice at busy airports and with recent research results (see Section 1). We define the following problems of efficiency maximization (EFF) and equity maximization (EQ), subject to the constraint that no more than $\hat{\lambda}_t$ flights may be scheduled during any period $t$

$$\begin{aligned}
\min\ & \sum_{i\in\mathcal{F}} |u_i| \quad \text{(EFF)}\\
\text{s.t. } & w_{it} \geq w_{i,t+1}, \quad \forall\, i\in\mathcal{F}, \forall\, t\in\mathcal{T},\\
& w_{i1} = 1, \quad \forall\, i\in\mathcal{F},\\
& \sum_{t\in\mathcal{T}} (w_{it} - S_{it}) = u_i, \quad \forall\, i\in\mathcal{F},\\
& \sum_{i\in\mathcal{F}} (w_{it} - w_{i,t+1}) \leq \hat{\lambda}_t, \quad \forall\, t\in\mathcal{T},\\
& |u_i| \leq \delta^*, \quad \forall\, i\in\mathcal{F};
\end{aligned}$$

$$\begin{aligned}
\operatorname{lex\,min}\ & \left(\frac{1}{|\mathcal{F}_a|}\sum_{i\in\mathcal{F}_a}|u_i|\right)_{a\in\mathcal{A}} \quad \text{(EQ)}\\
\text{s.t. } & w_{it} \geq w_{i,t+1}, \quad \forall\, i\in\mathcal{F}, \forall\, t\in\mathcal{T},\\
& w_{i1} = 1, \quad \forall\, i\in\mathcal{F},\\
& \sum_{t\in\mathcal{T}} (w_{it} - S_{it}) = u_i, \quad \forall\, i\in\mathcal{F},\\
& \sum_{i\in\mathcal{F}} (w_{it} - w_{i,t+1}) \leq \hat{\lambda}_t, \quad \forall\, t\in\mathcal{T},\\
& |u_i| \leq \delta^*, \quad \forall\, i\in\mathcal{F}.
\end{aligned}$$

Proposition 2 shows that efficiency and equity can be jointly maximized if the number of flights scheduled over any set of three consecutive time periods is lower than the total number of flights that can be scheduled over the same three periods. In that case, the scheduling interventions in the periods with more than $\hat{\lambda}_t$ flights scheduled can be treated independently and the problem can thus be reduced to a series of one-period problems.

**Proposition 2.** *If* $\sum_{l=t-1}^{t+1}|\mathcal{D}_l| \leq \sum_{l=t-1}^{t+1}\hat{\lambda}_l$ *for each period* $t\in\mathcal{T}$*, then there exists a solution that simultaneously solves* (EFF) *and* (EQ) *to optimality.*

**Proof.** Any feasible solution has to displace at least $(|\mathcal{D}_t| - \hat{\lambda}_t)^+$ flights in every period $t$, so $\Delta^* \geq \sum_{t\in\mathcal{T}}(|\mathcal{D}_t| - \hat{\lambda}_t)^+$. We first construct a feasible solution that reschedules exactly $\sum_{t\in\mathcal{T}}(|\mathcal{D}_t| - \hat{\lambda}_t)^+$ flights. To do so, we reschedule flights recursively from $t = 1, \ldots, T$, first to the preceding period (i.e., period $t-1$), up to capacity, and then to the following period (i.e., period $t+1$), if necessary. Specifically, we select a subset $\mathcal{K}_t^- \subset \mathcal{D}_t$ and then $\mathcal{K}_t^+ \subset \mathcal{D}_t \backslash \mathcal{K}_t^-$ such that

$$\begin{aligned}
|\mathcal{K}_t^-| &= \min\{\hat{\lambda}_{t-1} - (|\mathcal{D}_{t-1}| + |\mathcal{K}_{t-2}^+| - |\mathcal{K}_{t-1}^-| - |\mathcal{K}_{t-1}^+|),\\
&\qquad (|\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+|)^+\},\\
|\mathcal{K}_t^+| &= (|\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+| - |\mathcal{K}_t^-|)^+.
\end{aligned}$$

The subsets $\mathcal{K}_t^-$ and $\mathcal{K}_t^+$ are not uniquely determined, but we can choose any subsets of $\mathcal{D}_t$ that satisfy these properties. We define $u^{\text{eff}}$ as follows: $u_i^{\text{eff}} = -1$ for all $i\in\mathcal{K}_t^-$, $u_i^{\text{eff}} = +1$ for all $i\in\mathcal{K}_t^+$, and $u_i^{\text{eff}} = 0$ for all $i\notin \mathcal{K}_t^- \cup \mathcal{K}_t^+$. We define $w^{\text{eff}}$ accordingly (based on the constraints of (EFF)). According to Lemma 1 in the online appendix, the solution $(w^{\text{eff}}, u^{\text{eff}})$ is well defined, and, moreover, it is a feasible and optimal solution of (EFF). It reschedules exactly $(|\mathcal{D}_t| - \hat{\lambda}_t)^+$ flights from any period $t$ by one period each, so $\delta^* = 1$ (unless $|\mathcal{D}_t| \leq \hat{\lambda}_t$ for each period $t\in\mathcal{T}$, in which case $\delta^* = 0$), and $\Delta^* = \sum_{t\in\mathcal{T}}(|\mathcal{D}_t| - \hat{\lambda}_t)^+$.

We now denote by $(w^{\text{eq}}, u^{\text{eq}})$ an optimal solution of (EQ). Note that (EQ) has the same feasible region as (EFF); therefore, it admits a feasible solution, and, since it is discrete and bounded, it also admits an optimal solution. In Lemma 2 in the online appendix, we construct a solution $(w^*, u^*)$ such that $|\{i\in\mathcal{D}_t \mid u_i^* = -1\}| = |\mathcal{K}_t^-|$, $|\{i\in\mathcal{D}_t \mid u_i^* = +1\}| = |\mathcal{K}_t^+|$, and $|u_i^*| \leq |u_i^{\text{eq}}|$ for all $i\in\mathcal{F}$. The solution $(w^*, u^*)$ displaces exactly the same number of flights from each period $t$ to $t-1$ and to $t+1$ as does the solution $(w^{\text{eff}}, u^{\text{eff}})$, so it is an optimal solution of (EFF). Moreover, it only displaces flights that were displaced under solution $(w^{\text{eq}}, u^{\text{eq}})$, so we have $\sum_{i\in\mathcal{F}_a}|u_i^*| \leq \sum_{i\in\mathcal{F}_a}|u_i^{\text{eq}}|$ for each airline $a\in\mathcal{A}$. Therefore, $u^*$ is also an optimal solution of (EQ). □

Proposition 3 shows that efficiency and equity can be jointly maximized if each airline's share of flights is identical across all periods. Specifically, we assume that the number of flights scheduled by each airline $a$ during each period $t$ is the product of an airline-related factor $\alpha_a$ and a period-related factor $\beta_t$. In that case, there is significant flexibility in terms of choosing the airlines whose flights should be rescheduled, which enables equity maximization at no efficiency loss. For simplicity, we focus on the case of $\delta^* = 1$ period, which is also the most common case encountered with real-world data (see Section 5).

**Proposition 3.** *If* $\delta^* = 1$ *period and there exist integers* $(\alpha_a)_{a\in\mathcal{A}}$ *and* $(\beta_t)_{t\in\mathcal{T}}$ *such that* $|\mathcal{D}_t \cap \mathcal{F}_a| = \alpha_a\beta_t$ *for each airline* $a\in\mathcal{A}$ *and for each period* $t\in\mathcal{T}$*, then there exists a solution that simultaneously solves* (EFF) *and* (EQ) *to optimality.*

**Proof.** We consider an optimal solution of (EFF), denoted by $(w^{\text{eff}}, u^{\text{eff}})$. We denote by $X_t^+$ (respectively, $X_t^-$) the number of flights that, under solution $(w^{\text{eff}}, u^{\text{eff}})$, are displaced from period $t$ to period $t+1$ (respectively, $t-1$), that is, $X_t^+ = |\{i\in\mathcal{D}_t \mid u_i^{\text{eff}} = +1\}|$ (respectively, $X_t^- = |\{i\in\mathcal{D}_t \mid u_i^{\text{eff}} = -1\}|$). We also denote by $X_t$ the total number of flights displaced from period $t$, that is, $X_t = X_t^- + X_t^+$ for each $t\in\mathcal{T}$. The optimal objective value function of (EFF) is $\Delta^* = \sum_{i=1}^n |u_i^{\text{eff}}| = \sum_{t\in\mathcal{T}}(X_t^- + X_t^+) = \sum_{t\in\mathcal{T}} X_t$. We aim to construct a solution $(w^*, u^*)$ that is feasible, efficiency maximizing, and equity maximizing.

A sufficient condition for $(w^*, u^*)$ to be feasible and efficiency maximizing is to ensure that, for each period $t$, the number of flights rescheduled to $t-1$ and to $t+1$, respectively, under solution $(w^*, u^*)$ is equal to that under solution $(w^{\text{eff}}, u^{\text{eff}})$ for every period $t$, that is $|\{i \in \mathcal{D}_t \mid u_i^* = -1\}| = X_t^-$ and $|\{i \in \mathcal{D}_t \mid u_i^* = +1\}| = X_t^+$ for all $t \in \mathcal{T}$. Indeed, if this condition is satisfied, the aggregate schedule is identical under solutions $(w^{\text{eff}}, u^{\text{eff}})$ and $(w^*, u^*)$, that is, $\sum_{i\in\mathcal{F}}(w_{it}^{\text{eff}} - w_{i,t+1}^{\text{eff}}) = \sum_{i\in\mathcal{F}}(w_{it}^* - w_{i,t+1}^*)$ for all $t \in \mathcal{T}$. Therefore, solution $(w^*, u^*)$ is feasible. Moreover, under this condition, $\sum_{i\in\mathcal{D}_t}|u_i^*| = X_t^- + X_t^+$ for all $t \in \mathcal{T}$, and by summing over $t$ we obtain $\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{D}_t}|u_i^*| = \sum_{t\in\mathcal{T}} X_t$, that is, $\sum_{i\in\mathcal{F}}|u_i^*| = \Delta^*$, so solution $(w^*, u^*)$ solves (EFF).

A sufficient condition for a feasible and efficiency-maximizing solution $(w^*, u^*)$ to optimally solve (EQ) is to ensure that the vector $U$ defined by $U_a = \sum_{i\in\mathcal{F}_a}|u_i^*|$ for each airline $a \in \mathcal{A}$ solves the following problem, denoted by $\mathcal{P}(\Delta^*)$:

$$\begin{aligned} \operatorname{lex\,min}\ & \left(\frac{U_a}{|\mathcal{F}_a|}\right)_{a\in\mathcal{A}} \\ \text{s.t.}\ & \sum_{a\in\mathcal{A}} U_a \geq \Delta^*, \\ & U_a \geq 0, U_a \text{ integer.} \end{aligned}$$

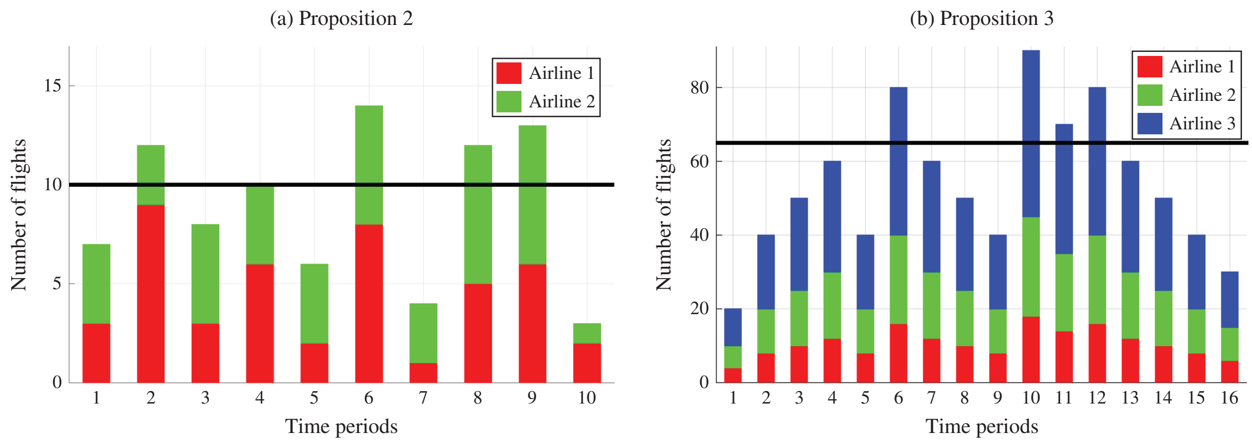
We construct an optimal solution of Problem $\mathcal{P}(\Delta^*)$ in the online appendix (Lemma 9), which we will use in this proof to construct a solution of (EQ). First, let us summarize how this optimal solution of Problem $\mathcal{P}(\Delta^*)$ is constructed. We assume without loss of generality that the greatest common divisor (gcd) of $(\alpha_a)_{a\in\mathcal{A}}$ is equal to 1 (if that is not the case then we can adjust the values of $(\alpha_a)_{a\in\mathcal{A}}$ and $(\beta_t)_{t\in\mathcal{T}}$ to ensure that this condition holds). Note that $|\mathcal{F}_a| = \alpha_a(\sum_{t\in\mathcal{T}}\beta_t)$ for all $a \in \mathcal{A}$ and thus $\gcd(|\mathcal{F}_a|)_{a\in\mathcal{A}} = \sum_{t\in\mathcal{T}}\beta_t$. We then have $|\mathcal{F}_a|/\gcd(|\mathcal{F}_a|)_{a\in\mathcal{A}} = \alpha_a$ for all $a \in \mathcal{A}$. We denote by $N = \sum_{a\in\mathcal{A}}\alpha_a$. Let $\mathbb{1}$ denote the indicator function. According to Lemma 9 (see the online appendix), there exists a sequence $(a_1, \ldots, a_N) \in \mathcal{A}^N$ such that $\sum_{i=1}^N \mathbb{1}(a_i = a) = \alpha_a$ for all $a \in \mathcal{A}$, and the $|\mathcal{A}|$-dimensional vector $U$ defined by $U_a = q\alpha_a + \sum_{i=1}^r \mathbb{1}(a_i = a)$ for all $a \in \mathcal{A}$ is an optimal solution of $\mathcal{P}(\Delta^*)$, where integers $q$ and $r$ denote the quotient and the remainder of the Euclidean division of $\Delta^*$ by $N$ (i.e., $\Delta^* = qN + r$, $0 \leq q$, and $0 \leq r < N$). We denote by $\Psi$ the sequence $\Psi = (a_1, \ldots, a_N, \ldots, a_1, \ldots, a_N, a_1, \ldots, a_r)$, where the full sequence $(a_1, \ldots, a_N)$ is repeated $q$ times. By construction, $\sum_{i=1}^{\Delta^*}\mathbb{1}(\Psi_i = a) = q\alpha_a + \sum_{i=1}^r \mathbb{1}(a_i = a)$ for all $a \in \mathcal{A}$ and thus the vector $U$ defined by $U_a = \sum_{i=1}^{\Delta^*}\mathbb{1}(\Psi_i = a)$ for all $a \in \mathcal{A}$ is an optimal solution of $\mathcal{P}(\Delta^*)$.

We now construct a solution $(w^*, u^*)$ that satisfies (i) the sufficient conditions for feasibility and for efficiency maximization: $|\{i \in \mathcal{D}_t \mid u_i^* = -1\}| = X_t^-$ for all $t \in \mathcal{T}$, and $|\{i \in \mathcal{D}_t \mid u_i^* = +1\}| = X_t^+$ for all $t \in \mathcal{T}$, as well as (ii) the sufficient condition for equity maximization: $\sum_{i\in\mathcal{F}_a}|u_i^*| = \sum_{i=1}^{\Delta^*}\mathbb{1}(\Psi_i = a)$ for all $a \in \mathcal{A}$. To do so, we construct a solution that displaces one flight from airline $\Psi_1$ in period 1, then one flight from airline $\Psi_2$ in period $1, \ldots,$ then one flight from airline $\Psi_{X_1}$ in period 1, then one flight from airline $\Psi_{X_1+1}$ in period 2, then one flight from airline $\Psi_{X_1+2}$ in period $2, \ldots,$ then one flight from airline $\Psi_{X_1+X_2}$ in period $2, \ldots,$ and finally one flight from airline $\Psi_{X_1+\cdots+X_T}$ in period $T$ (of course, each airline may be repeated several times in each sequence). We denote by $y_t$ the total number of flights displaced from period 1 through $t-1$ (both inclusive), that is, $y_t = \sum_{s=1}^{t-1} X_s$. Note that $y_1 = 0$ and $y_{T+1} = \sum_{s\in\mathcal{T}} X_s = \Delta^*$. We denote by $V_{at}$ the number of times airline $a$ is repeated in the $X_t$ indices between $y_t + 1$ and $y_{t+1}$ (both inclusive), that is, $V_{at} = \sum_{i=y_t+1}^{y_{t+1}}\mathbb{1}(\Psi_i = a)$ for all $a \in \mathcal{A}$. Given the periodicity of the sequence $\Psi$, any consecutive set of $N\beta_t$ values of $\Psi_i$ includes exactly $\alpha_a\beta_t$ elements equal to $a$, for all $a \in \mathcal{A}$. Since $y_{t+1} - y_t \leq N\beta_t$, we have $V_{at} \leq \alpha_a\beta_t$ for all $a \in \mathcal{A}$ and $t \in \mathcal{T}$. We can thus define a set $\mathcal{J}_{at} \subseteq \mathcal{D}_t \cap \mathcal{F}_a$ such that $|\mathcal{J}_{at}| = V_{at}$. As in the proof of Proposition 2, $\mathcal{J}_{at}$ is not uniquely determined, but we can choose any subset of $\mathcal{D}_t \cap \mathcal{F}_a$ that satisfies this property. We construct a solution that displaces the flights in the sets $\mathcal{J}_{at}$ such that the number of flights rescheduled to period $t-1$ (respectively, $t+1$) is equal to $X_t^-$ (respectively, $X_t^+$). For each $t \in \mathcal{T}$, we partition $\bigcup_{a\in\mathcal{A}}\mathcal{J}_{at}$ into two subsets $\mathcal{K}_t^+$ and $\mathcal{K}_t^-$ such that $|\mathcal{K}_t^+| = X_t^+$ and $|\mathcal{K}_t^-| = X_t^-$. We then define (i) $u_i^* = -1$ for all $i \in \mathcal{K}_t^-$, (ii) $u_i^* = +1$ for all $i \in \mathcal{K}_t^+$, (iii) $u_i^* = 0$ for all $i \notin (\mathcal{K}_t^- \cup \mathcal{K}_t^+)$. We define $w^*$ accordingly (based on the constraints of (EFF) and (EQ)).

By construction, the solution $(w^*, u^*)$ satisfies the sufficient conditions for feasibility and efficiency maximization, so it solves Problem (EFF) to optimality. Moreover, we have $\sum_{i\in\mathcal{D}_t\cap\mathcal{F}_a}|u_i^*| = V_{at}$ for all $a \in \mathcal{A}$ and for all $t \in \mathcal{T}$. By summing over $t \in \mathcal{T}$, we obtain $\sum_{i\in\mathcal{F}_a}|u_i^*| = \sum_{t\in\mathcal{T}}\sum_{i=y_t+1}^{y_{t+1}}\mathbb{1}(\Psi_i = a)$, that is $\sum_{i\in\mathcal{F}_a}|u_i^*| = \sum_{i=y_1+1}^{y_{T+1}}\mathbb{1}(\Psi_i = a) = \sum_{i=1}^{\Delta^*}\mathbb{1}(\Psi_i = a)$. Therefore, the solution $(w^*, u^*)$ solves Problem (EQ). □

In summary, in the case where scheduling interventions are based only on temporal shifts in demand, efficiency and equity can be jointly maximized if (i) no network connections need to be maintained, (ii) all flights are equally valued, and (iii) airline schedules of flights satisfy the conditions of Propositions 2 or 3 (or both). Examples of these conditions are shown in Figure 3. Under the conditions of Proposition 2 (Figure 3(a)), the imbalances between demand and capacity are small enough so no time period is such that some flights get displaced *to* that period and some other flights get displaced *from* that period (as shown in Proposition 2). Under the conditions of Proposition 3 (Figure 3(b)), the schedules of flights of all of the airlines exhibit the same intraday variations. Even though these conditions are not exactly satisfied in practice, our computational experiments reported in Section 5 show that the

**Figure 3.** (Color online) Example of Scheduling Conditions of Propositions 2 and 3




insights derived in these two cases can be applicable in practical settings.

## 4.2. Instances of Efficiency/Equity Trade-Off

Based on the discussion above, in the case where the scheduling interventions are based on temporal shifts in demand, a trade-off between efficiency and equity might arise through (i) interairline variations in intraday flight schedule patterns (we refer to it simply by "differentiated airline schedules") when the conditions of Proposition 3 are not satisfied; (ii) network connections; and (iii) intra-airline variations in flight valuations (we refer to it simply by "differentiated flight valuations"). Note that, in the case where the scheduling interventions are based only on reductions in demand, a trade-off between efficiency and equity might also arise through network connections, and differentiated flight valuations, which can be similarly demonstrated through examples.

We first provide an example that shows that weighted efficiency and equity may not be jointly maximized in the presence of differentiated airline schedules. Figure 4 shows a hypothetical example with an unconstrained schedule in a seven-period case with two airlines and 26 flights per airline, and a capacity constraint (as a simplified representation of the on-time performance constraints) that ensures that no more than 10 flights may be scheduled per period. We assume that all flights are valued equally and that there are no connections. We also assume that airline 1's flights (shown in red) are concentrated at earlier periods, and airline 2's flights (shown in green) are concentrated at later periods. Note that the conditions of either Proposition 2 or 3 are not satisfied here. Figure 4(a) (respectively, Figure 4(b)) shows which flights are rescheduled to later or earlier times for an efficiency-maximizing solution (respectively, an equity-maximizing solution). Since the capacity constraint is only violated during period 5, when all flights scheduled are airline 1's flights, every efficiency-maximizing solution displaces four flights from airline 1 to later times, by one period each (one such efficiency-maximizing solution is shown by "+1"s in Figure 4(a)). The resulting total displacement is equal to four periods, and the airline disutilities are equal to 4/26 for airline 1 and 0 for airline 2. By contrast, every equity-maximizing solution displaces three flights of

**Figure 4.** (Color online) Trade-Off Between Weighted Efficiency and Equity Because of Differentiated Airline Schedules

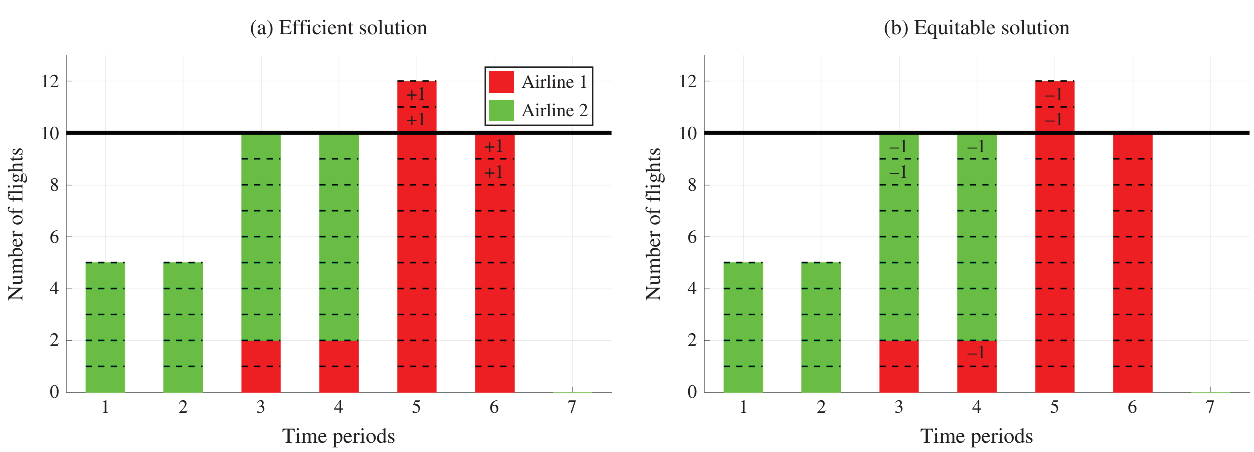
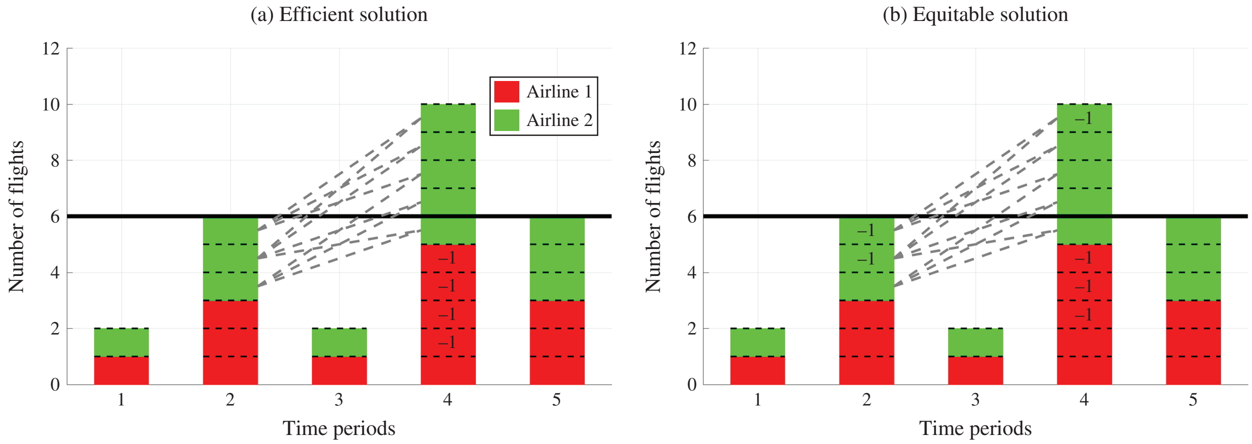


**Figure 5.** (Color online) Trade-Off Between Weighted Efficiency and Equity Because of Network Connections

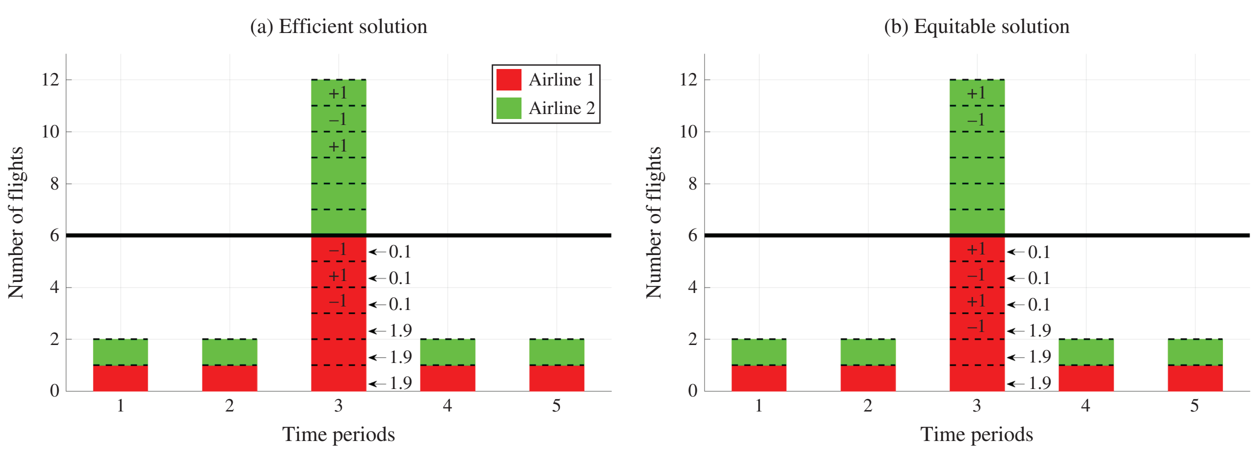



each to earlier times, by one period each (one such equity-maximizing solution is shown by "−1"s in Figure 4(b)). The resulting total displacement is equal to six periods, and each airline's disutility is equal to 3/26. In turn, the set of efficiency-maximizing solutions and the set of equity-maximizing solutions have no overlap.

We now provide an example that shows that weighted efficiency and equity may not be jointly optimized in the presence of network connections. Intuitively, if one airline's network is significantly more connected than another airline's, then the former airline's flights are likely to be more difficult to reschedule. In turn, maximizing efficiency may involve assigning more displacement to the latter airline's flights rather than the former's, at some equity loss. Figure 5 shows such an example with five periods, two airlines with 13 flights each, and a capacity of six flights per period. Note that the conditions of both Propositions 2 and 3 would be satisfied in the absence of network connections. But airline 2's network involves a number of connections, whereas airline 1's network has no connections. We represent connections by dashed, gray "links" between flight pairs, and we assume that each connection requires a two-period interval between the flights in the connection at a minimum. In this case, every efficiency-maximizing solution displaces four of airline 1's flights (the airline with no connections) by one period each. The resulting total displacement is equal to four periods, and the airline disutilities are equal to 4/13 for airline 1 and 0 for airline 2. By contrast, every equity-maximizing solution displaces three flights of each airline, by one period each. The resulting total displacement is equal to six periods, and each airline's disutility is equal to 3/13. The set of efficiency-maximizing solutions and the set of equity-maximizing solutions have no overlap.

Finally, we provide an example that shows that weighted efficiency and equity may not be jointly optimized in the presence of differentiated flight valuations. Figure 6 shows an example with five periods, two airlines with 10 flights each, and a capacity of six flights per period. Note that the conditions of both Propositions 2 and 3 would be satisfied under uniform flight valuations. Yet we assume that every flight, except the six flights scheduled by airline 1 in period 3, has a value equal to $v_i = 1$, and, among the

**Figure 6.** (Color online) Trade-Off Between Weighted Efficiency and Equity Because of Differentiated Flight Valuations



remaining six flights, three have a value of $v_i = 0.1$ each, and three others have a value of $v_i = 1.9$ each (as a result, the average value of airline 1's flights is equal to 1). Every efficiency-maximizing solution displaces the three flights of value $v_i = 0.1$ and three flights of value $v_i = 1$ from period 3. The optimal value of the weighted displacement is equal to 3.3 and the airline disutilities are equal to 0.3/10 for airline 1 and to 3/10 for airline 2. By contrast, every equity-maximizing solution displaces four flights of airline 1 and two flights of airline 2. The weighted displacement is equal to 4.2 and the airline disutilities are equal to 2.2/10 for airline 1 and to 2/10 for airline 2. The set of efficiency-maximizing solutions and the set of equity-maximizing solutions have no overlap.

## 5. Computational Results

We implement the models developed in Section 3 for a case study at JFK Airport. We show that, in realistic instances, interairline equity can be significantly improved at no (or minimal) efficiency losses if flights are equally valued. We then show that, even under differentiated flight valuations, the price of equity is consistently significantly smaller than the price of efficiency, that is, that significant equity gains can be obtained at small losses in efficiency.

### 5.1. Experimental Setup

We implement our models at John F. Kennedy Airport, one of the most congested airports in the United States, and with a peaked schedule of flights that offers opportunities for delay reductions through scheduling interventions. We consider data from 2007, as no scheduling interventions were in place at JFK that year. Specifically, we use data from the nine days that correspond to the nine deciles of the distribution of the number of daily flights at JFK in 2007, to capture the variability of flight schedules throughout the year. We focus on September 18, 2007 as a "representative" day, because the number of flights scheduled on September 18 equals the median of the number of daily flights at JFK in 2007. Estimates of JFK's capacity in various operating conditions were obtained from Simaiakis (2012). Flight schedules were obtained from the Aviation System Performance Metrics (ASPM) database (Federal Aviation Administration 2013). We group partner airlines together, as major airlines typically coordinate planning and scheduling decisions with their subsidiaries, and passengers can easily connect between flights operated by partner airlines. Specifically, we consider four groups of airlines: (i) Delta Airlines (DAL) and its regional partners (which operated a total of 320 flights on September 18, 2007 at JFK); (ii) American Airlines (AAL) and its regional partners (260 flights); (iii) JetBlue Airways (JBU) (174 flights); and (iv) all other airlines, each of which represents a smaller share of traffic at JFK (408 flights combined). These scheduling data were used to construct sets $\mathcal{F}$, $\mathcal{F}^{\text{arr}}$, $\mathcal{F}^{\text{dep}}$, $\mathcal{F}_a$, $\mathcal{S}^{\text{arr}}$, and $\mathcal{S}^{\text{dep}}$.

We reconstructed aircraft and passenger connections to determine $\mathcal{C}$, $t^{\min}$, and $t^{\max}$. Aircraft connections were obtained from the ASPM database (Federal Aviation Administration 2013). We use the minimum aircraft turnaround time between any pair of flights estimated by Pyrgiotis (2011) as a function of the aircraft type, of the airline, and of whether the airport is a hub airport for the airline or not. We use a maximum turnaround time equal to the planned turnaround time plus 15 minutes to maintain comparable aircraft utilization. We obtained passenger connections data from a database developed by Barnhart, Fearing, and Vaze (2014), based on a discrete choice model for estimating historical passenger flows. We estimate the minimum passenger connection time at JFK as the 5th percentile of the distribution of all planned passenger connection times. Because of data unavailability, we do not reconstruct crew connections here, but their consideration could be easily added as estimates of historical crew schedules become available (Wei and Vaze 2018).

With the actual schedule of flights on September 18, 2007, the peak expected arrival and departure queue lengths are equal to $\max_{t\in\mathcal{T}} E(A_t) = 14.6$ aircraft and $\max_{t\in\mathcal{T}} E(D_t) = 28.1$, respectively—obtained using the model of airport congestion shown in Equation (9). We vary the expected arrival queue length target $A_{\text{MAX}}$ from 15 to 11 aircraft, and the expected departure queue length target $D_{\text{MAX}}$ from 23 to 15 aircraft. These targets can be met under scheduling interventions restricted only to temporal shifts in demand and without imposing prohibitively large flight displacements.

We implemented the integer programming models in GAMS 24.0 using CPLEX 12.5 and the dynamic programming model and the stochastic queuing model in MATLAB 8.1. We attempted to find solutions to the integer programming models within an optimality gap of 1%. If none was found after 30 minutes, we accepted the best solution obtained till that time.

### 5.2. Uniform Flight Valuations

We first consider the case where all flights are equally valued, that is, $v_i = 1$ for all $i \in \mathcal{F}$. This corresponds to current practice, where the airlines do not provide any inputs on relative timetabling flexibility of their flights, and scheduling interventions are performed under the "a flight is a flight" paradigm. We first describe the computational performance of our modeling approach, including a comparison of Problems P3($\rho$) and $\widehat{\text{P3}(\rho)}$, and then present computational results that show that equity can be maximized at no (or small) efficiency losses in the case of uniform valuations.

**Table 2.** Computational Performance of Full and Simplified Equity-Maximization Algorithms

| | | $A_{MAX}=15$, $D_{MAX}=20$ | | | $A_{MAX}=11$, $D_{MAX}=15$ | | |
|---|---|---|---|---|---|---|---|
| | Metric | 02/04 | 09/18 | 05/25 | 02/04 | 09/18 | 05/25 |
| Model | Number of flights $\lvert\mathcal{F}\rvert$ | 1,465 | 1,592 | 1,678 | 1,465 | 1,592 | 1,678 |
| P1, P2 | # IP | 10 | 10 | 11 | 11 | 10 | 14 |
| | Avg. IP CPU (s) | 29 | 32 | 222 | 29 | 45 | 1,318 |
| | Avg. IP gap (%) | 0.9 | 0.7 | 0.9 | 0.9 | 1.0 | 9.6 |
| | # DP/SQM | 10 | 10 | 11 | 11 | 10 | 14 |
| | Avg. DP/SQM CPU (s) | 70 | 69 | 69 | 69 | 69 | 69 |
| | Total CPU (min) | 17 | 17 | 52 | 18 | 87 | 324 |
| P3(0) | # IP | 29 | 28 | 28 | 28 | 29 | 28 |
| | Avg. IP CPU (s) | 16 | 17 | 22 | 31 | 18 | 339 |
| | Avg. IP gap (%) | 0 | 0 | 0 | 0.4 | 0 | 7.94 |
| | # DP/SQM | 1 | 2 | 2 | 12 | 2 | 5 |
| | Avg. DP/SQM CPU (s) | 71 | 69 | 71 | 69 | 70 | 76 |
| | Total CPU (min) | 9 | 10 | 13 | 28 | 11 | 164 |
| | $\Phi^*(0)$ (%) | 3.20 | 3.08 | 7.48 | 11.47 | 17.82 | 30.28 |
| $\widehat{\text{P3(0)}}$ | # IP | 4 | 4 | 4 | 4 | 4 | 4 |
| | Avg. IP CPU (s) | 24 | 23 | 24 | 21 | 28 | 43 |
| | Avg. gap (%) | 0 | 0.2 | 0.4 | 0.3 | 0.3 | 0.3 |
| | Total CPU (min) | 1.6 | 1.5 | 1.6 | 1.4 | 1.8 | 2.9 |
| | $\Phi^*(0)$ (%) | 3.39 | 3.08 | 7.48 | 11.47 | 17.82 | 31.07 |

**5.2.1. Computational Performance.** Table 2 reports, for three days of operations (corresponding to the first, fifth, and ninth deciles of the daily flight distribution in 2007) and for two sets of on-time performance targets $A_{MAX}$ and $D_{MAX}$, the computational performance of the efficiency-maximizing problem (Problems P1 and P2) and of the full and simplified equity-maximizing problems under optimal efficiency (Problems P3(0)) and $\widehat{\text{P3(0)}}$, respectively). It also shows the largest airline disutility $\Phi^*(0)$ (i.e., largest average per-flight displacement, expressed as a percentage) resulting from Problems P3(0) and $\widehat{\text{P3(0)}}$. First, note that the efficiency-maximizing problem involves 10–15 iterations, each one consisting of one integer programming model, one dynamic programming model, and one stochastic queuing model. The computational time requirement of each integer programming model increases with the stringency of the on-time performance targets $A_{MAX}$ and $D_{MAX}$ and with the number of flights scheduled during the day. Overall, the efficiency-maximizing solution is found within 15 minutes to a few hours (Jacquillat and Odoni 2015a).

Second, Problem P3(0) also involves a similar iterative algorithm for every single airline in the lexicographic minimization, which leads to a total of nearly 30 integer-programming models (with four airlines). A number of them result in infeasible solutions, which explains the fact that fewer iterations involve actually solving the dynamic programming model and stochastic queuing model. Nonetheless, the full solution algorithm terminates in 9 to 164 minutes. By contrast, the simplified approach to equity maximization involves only $\lvert\mathcal{A}\rvert$ (here, 4) integer programming models, and terminates in 1.4 to 2.9 minutes (that is, an improvement by a factor of 5.6 to 56.6, as compared to Problem P3(0). Finally, the largest airline disutility values $\Phi^*(0)$ are very similar after Problems P3(0) and $\widehat{\text{P3(0)}}$. They are, in fact, identical in all but two instances, and do not differ by more than 0.8% in any of these six instances. This suggests that solving Problem $\widehat{\text{P3(0)}}$ provides a much faster solution approach than solving Problem P3($\rho$), while yielding similar levels of interairline equity. Therefore, Problems $\widehat{\text{P3}(\rho)}$ provide a much more tractable solution architecture than Problems P3($\rho$) in practical instances where the problem has to be combined with Problems P1 and P2, and has to be solved for the busiest days of the season and for multiple values of $\rho$ to determine the full Pareto frontier between efficiency and equity. Therefore the remainder of the results shown in this paper are based on Problems P1, P2, and $\widehat{\text{P3}(\rho)}$ (see Figure 2).

**5.2.2. Results.** We now compare the results obtained under an efficiency-maximization objective (Problems P1 and P2) to those obtained with interairline equity objectives (Problems $\widehat{\text{P3}(\rho)}$). This comparison shows the extent to which interairline equity can be achieved in scheduling interventions under current scheduling conditions and uniform flight valuations.

Note that the solution of Problem P2 is arbitrarily "chosen" by the optimization solver from the set of (possibly) multiple optimal solutions. To characterize the equity range among efficiency-maximizing solutions, we also determine the solution that *minimizes* interairline equity, that is, which lexicographically *maximizes* airline disutilities, while ensuring the optimal value of efficiency. This characterizes the efficiency-maximizing solution that performs the worst in terms of interairline equity. We denote this problem by $\overline{\text{P2}}$. In the remainder of this paper, we will show four solutions: (i) the solution that maximizes efficiency, and minimizes equity among all solutions that maximize efficiency (i.e., solution to Problem $\overline{\text{P2}}$), referred to as the "EFFMAX-EQMIN" solution henceforth; (ii) the solution that maximizes efficiency with no equity consideration (i.e., solution to Problem P2), referred to as the "EFFMAX" solution henceforth; (iii) the solution that maximizes efficiency and maximizes equity among all solutions that maximize efficiency (i.e., solution to Problem $\widehat{\text{P3}(0)}$), referred to as the "EFFMAX-EQMAX" solution henceforth; (iv) the solution that maximizes equity (i.e., solution to Problem $\widehat{\text{P3}(\rho*)}$), referred to as the "EQMAX" solution henceforth.

Table 3 shows, for different sets of on-time performance targets $A_{\text{MAX}}$ and $D_{\text{MAX}}$, the total schedule displacement faced by each airline, and each airline's disutility (i.e., its average per-flight displacement) for the EFFMAX, EFFMAX-EQMIN, and EQMAX solutions (i.e., for Problems P2, $\overline{\text{P2}}$, and $\widehat{\text{P3}(\rho*)}$, respectively). It also reports the ratio of the largest to smallest airline disutility $\max_a \sigma_a^*/(\min_a \sigma_a^*)$. As $A_{\text{MAX}}$ and $D_{\text{MAX}}$ become smaller, the resulting schedule displacement increases (Jacquillat and Odoni 2015a), but these results show that, for any set of values of $A_{\text{MAX}}$ and $D_{\text{MAX}}$ considered, the modeling approach developed in this paper provides strong equity gains at no loss in efficiency. Note, first, that the EFFMAX-EQMIN solution results in max–min ratios $\max_a \sigma_a/(\min_a \sigma_a)$ ranging between 7 and 45. For the cases considered, AAL and JBU tend to be much more significantly penalized than DAL, which is reflected through more of their flights being displaced and through much higher disutility values. The set of efficiency-maximizing solutions thus contains some highly inequitable outcomes. The EFFMAX solution does not result in the most inequitable outcome in that set, but provides solutions that still impact some airlines (here, AAL, JBU, and the "other" airlines) more negatively than others (here, DAL). Interairline equity is achieved only with the EQMAX solution. In that case, airline disutilities are much closer to each other than those obtained in the EFFMAX-EQMIN and EFFMAX solutions. Note that the differences in airlines' schedules of flights and network connectivities result in all four airlines *not* having the exact same disutility, but differences are very small (i.e., the max–min ratio $\max_a \sigma_a/(\min_a \sigma_a)$ is very close to 1) under the EQMAX solution. Most importantly, the EQMAX solution results in the same total displacement as the EFFMAX solution in all cases. Only the distribution of schedule displacements across the airlines is modified. In other words, efficiency and equity can be jointly maximized, and the price of equity ($\rho^*$) and the price of efficiency are both zero.

We now extend the results to each of the nine days described in our experimental setup (by increasing number of flights at JFK). Table 4 reports, for two sets of on-time performance targets $A_{\text{MAX}}$ and $D_{\text{MAX}}$, the optimal min–max and weighted displacements ($\delta^*$ and $\Delta^*$), the prices of equity and efficiency ($P_{\text{eq}}$ and $P_{\text{eff}}$), as well as the ratios of the largest to smallest airline disutilities $\max_a \sigma_a^*/(\min_a \sigma_a^*)$ under the EFFMAX-EQMIN solution (Problem $\overline{\text{P2}}$), the EFFMAX solution (Problem P2), and the EQMAX solution (Problem $\widehat{\text{P3}(\rho*)}$). Note, first,

**Table 3.** Results for September 18, 2007 Under Uniform Flight Valuations, for Different On-Time Performance Targets

| Targets | | | Displacement | | | | | Disutility: $\sigma_a = \frac{1}{\lvert\mathcal{F}_a\rvert}\sum_{i\in\mathcal{F}_a}\lvert u_i\rvert$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_{\text{MAX}}$ | $D_{\text{MAX}}$ | Model | DAL | AAL | JBU | Others | All | DAL (%) | AAL (%) | JBU (%) | Others (%) | $\frac{\max_a \sigma_a^*}{\min_a \sigma_a^*}$ |
| 14 | 23 | $\overline{\text{P2}}$ | 1 | 13 | 1 | 5 | 20 | 0.3 | 5.0 | 0.6 | 1.2 | 16.00 |
| | | P2 | 1 | 8 | 2 | 9 | 20 | 0.3 | 3.1 | 1.1 | 2.2 | 9.85 |
| | | $\widehat{\text{P3}(\rho*)}$ | 4 | 5 | 3 | 8 | 20 | 1.3 | 1.9 | 1.7 | 2.0 | 1.57 |
| 13 | 20 | $\overline{\text{P2}}$ | 1 | 27 | 9 | 8 | 45 | 0.3 | 10.4 | 5.2 | 2.0 | 33.23 |
| | | P2 | 2 | 13 | 9 | 21 | 45 | 0.6 | 5.0 | 5.2 | 5.2 | 8.28 |
| | | $\widehat{\text{P3}(\rho*)}$ | 12 | 10 | 7 | 16 | 45 | 3.8 | 3.9 | 4.0 | 3.9 | 1.07 |
| 12 | 18 | $\overline{\text{P2}}$ | 1 | 36 | 17 | 10 | 64 | 0.3 | 13.9 | 9.8 | 2.5 | 44.31 |
| | | P2 | 4 | 18 | 14 | 28 | 64 | 1.3 | 6.9 | 8.1 | 6.9 | 6.44 |
| | | $\widehat{\text{P3}(\rho*)}$ | 18 | 14 | 10 | 22 | 64 | 5.3 | 5.8 | 5.8 | 5.4 | 1.07 |
| 11 | 15 | $\overline{\text{P2}}$ | 16 | 93 | 47 | 50 | 206 | 5.0 | 35.8 | 27.0 | 12.3 | 7.15 |
| | | P2 | 27 | 51 | 38 | 90 | 206 | 8.4 | 19.6 | 21.8 | 22.0 | 2.61 |
| | | $\widehat{\text{P3}(\rho*)}$ | 57 | 46 | 31 | 72 | 206 | 17.8 | 17.7 | 17.8 | 17.7 | 1.01 |

**Table 4.** Results for Each Day Considered Under Uniform Flight Valuations

| | | $A_{\text{MAX}} = 15, D_{\text{MAX}} = 20$ | | | | | | | $A_{\text{MAX}} = 11, D_{\text{MAX}} = 15$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Disp. | | Trade-off | | $\frac{\max_a \sigma_a^*}{\min_a \sigma_a^*}$ | | | Disp. | | Trade-off | | $\frac{\max_a \sigma_a^*}{\min_a \sigma_a^*}$ | | |
| Day | $\lvert\mathcal{F}^{\text{arr}} \cup \mathcal{F}^{\text{dep}}\rvert$ | $\delta^*$ | $\Delta^*$ | $P_{\text{eq}}$ (%) | $P_{\text{eff}}$ (%) | $\overline{\text{P2}}$ | P2 | $\widehat{\text{P3}(\rho*)}$ | $\delta^*$ | $\Delta^*$ | $P_{\text{eq}}$ (%) | $P_{\text{eff}}$ (%) | $\overline{\text{P2}}$ | P2 | $\widehat{\text{P3}(\rho*)}$ |
| 02/04 | 1,070 | 1 | 32 | 6.2 | 2.7 | ∞ | 4.33 | 1.17 | 1 | 116 | 1.7 | 1.5 | 5.16 | 2.72 | 1.12 |
| 01/10 | 1,112 | 1 | 28 | 0 | 0 | ∞ | 5.67 | 1.19 | 1 | 70 | 0 | 0 | 6.43 | 4.22 | 1.09 |
| 02/08 | 1,137 | 1 | 25 | 0 | 0 | ∞ | 8.00 | 1.09 | 1 | 92 | 0 | 0 | 8.38 | 5.11 | 1.17 |
| 01/25 | 1,153 | 1 | 33 | 0 | 0 | ∞ | ∞ | 1.09 | 1 | 105 | 3.8 | 5.1 | 9.71 | 6.33 | 1.04 |
| 09/18 | 1,162 | 1 | 34 | 0 | 0 | 29.54 | 19.00 | 1.09 | 1 | 206 | 0 | 0 | 7.15 | 2.61 | 1.01 |
| 10/15 | 1,177 | 1 | 41 | 0 | 0 | ∞ | ∞ | 1.08 | 1 | 194 | 0 | 0 | 3.96 | 2.97 | 1.01 |
| 06/01 | 1,192 | 1 | 39 | 0 | 0 | ∞ | 24.00 | 1.08 | 1 | 130 | 1.5 | 0.4 | 9.69 | 4.69 | 1.07 |
| 07/07 | 1,212 | 2 | 131 | 0 | 0 | 19.17 | 2.82 | 1.04 | 2 | 303 | 5.3 | 7.9 | 10.14 | 2.33 | 1.06 |
| 05/25 | 1,229 | 1 | 91 | 0 | 0 | 14.02 | 7.33 | 1.06 | 2 | 374 | 1.3 | 3.8 | 6.53 | 3.76 | 1.04 |

that the optimal displacement varies significantly as a function of the original schedule. Overall, the more flights scheduled in a day, the larger the resulting displacement, although the relationship is not strictly increasing because of intraday scheduling variations (Jacquillat and Odoni 2015a). Second, in 12 of the 18 cases reported, the price of equity and the price of efficiency are both equal to zero, that is, equity and efficiency can be jointly maximized. Across the remaining six cases, the prices of equity and efficiency are small (with average values of 3.3% and 3.6%, respectively), so interairline equity can be guaranteed with a small increase in the schedule displacement. Third, the set of efficiency-maximizing solutions includes highly inequitable outcomes for all days, reflected in the large values of the ratio $\max_a \sigma_a^*/(\min_a \sigma_a^*)$ for Problem $\overline{\text{P2}}$ and P2 (the ∞ values correspond to instances where the solutions displace no flight from at least one airline, so $\min_a \sigma_a^* = 0$), while the equitable outcomes result in similar disutility values among the four airlines.

Therefore, joint optimization (or near optimization) of efficiency and equity is generally achievable under current schedules of flights and uniform flight valuations (which is the assumption widely used in current practice). In light of the results from Section 4, this suggests that the effects of interairline variations in flight schedules and network connectivities are relatively weak and do not create, by themselves, a strong trade-off between efficiency and equity. This is because of the fact that peak-hour schedules typically include flights from several airlines and the schedules of all airlines exhibit network connections to some extent. So the situations depicted in Figures 4 and 5 are not typical of actual scheduling patterns at busy airports. Under these conditions reflecting actual scheduling patterns at busy airports, incorporating interairline equity objectives in scheduling interventions can thus yield significant benefits by balancing scheduling adjustments more fairly among the airlines at no (or small) efficiency losses.

### 5.3. Differentiated Flight Valuations

We now consider the case where all flights are not equally valued, and compare the outcomes of scheduling interventions when only the efficiency objectives are considered for the outcomes when equity objectives are also considered. This captures potential extensions of existing and other previously proposed mechanisms for airport scheduling interventions that would allow the airlines to provide the relative timetabling flexibility of their flights (e.g., auction, credit-based mechanism). Since the flight valuations rely on information that is often private to the airlines and since they are challenging to estimate using available public data, we use two different types of approximate approaches to estimate their impact on our efficiency-equity trade-off results. We first consider the case where the average flight valuation is identical for all airlines, to identify the impact of the *intra-airline distribution* of flights valuations. We then consider the more general case where average flight valuations may vary across the airlines, by approximating them using revenue estimates. All of the results reported in this section are obtained with the schedule from September 18, 2007.

We first sample $(v_i)_{i\in\mathcal{F}}$ by keeping the average flight valuation of all airlines equal to 1 (without loss of generality), and varying the distribution of flight valuations for one given airline $a$. We set $v_i = 1$ for all $i \notin \mathcal{F}_a$. We partition the set of flights $\mathcal{F}_a$ of airline $a$ into two subsets $\mathcal{F}_a^{(1)}$ and $\mathcal{F}_a^{(2)}$ such that $\mathcal{F}_a^{(1)} \cap \mathcal{F}_a^{(2)} = \varnothing$ and $\mathcal{F}_a^{(1)} \cup \mathcal{F}_a^{(2)} = \mathcal{F}_a$. We can think of $\mathcal{F}_a^{(1)}$ (respectively, $\mathcal{F}_a^{(2)}$) as the set of the more flexible flights (respectively, the less flexible flights) of airline $a$. We choose to represent the valuations of the flights in $\mathcal{F}_a^{(1)}$ (respectively, $\mathcal{F}_a^{(2)}$) by a gamma distribution $\Gamma_1(\mu_1, k)$ (respectively, $\Gamma_2(\mu_2, k)$) with mean $\mu_1$ (respectively, $\mu_2$) and shape parameter $k$, with $\mu_1 < \mu_2$. We adjust the shape parameter of these distributions such that the 95th percentile of the former distribution coincides with the

5th percentile of the latter. These choices of distributions and parameters are made to provide a transparent and flexible bimodal characterization of flight valuations such that the valuations of flights in $\mathcal{F}_a^{(1)}$ are, in most cases, lower than the valuations of flights in $\mathcal{F}_a^{(2)}$. Finally, we set the values of flights in $\mathcal{F}_a^{(1)}$ (respectively, $\mathcal{F}_a^{(2)}$) equal to $\Theta_1^{-1}(1/(|\mathcal{F}_a^{(1)}|+1))$, $\Theta_1^{-1}(2/(|\mathcal{F}_a^{(1)}|+1))$, $\ldots, \Theta_1^{-1}(|\mathcal{F}_a^{(1)}|/(|\mathcal{F}_a^{(1)}|+1))$ (respectively, $\Theta_2^{-1}(1/|\mathcal{F}_a^{(2)}|+1)$, $\Theta_2^{-1}(2/(|\mathcal{F}_a^{(2)}|+1)), \ldots, \Theta_2^{-1}(|\mathcal{F}_a^{(2)}|/(|\mathcal{F}_a^{(2)}|+1)))$, where $\Theta_1$ (respectively, $\Theta_2$) denotes the cumulative distribution function of $\Gamma_1(\mu_1, k)$ (respectively, $\Gamma_2(\mu_2, k)$). This sampling strategy ensures that the resulting set of flight valuations is distributed "smoothly" across the distributions being considered without sampling these values multiple times. For each airline, we vary two parameters: (i) the fraction of flights in $\mathcal{F}_a^{(1)}$, denoted by $\eta = |\mathcal{F}_a^{(1)}|/|\mathcal{F}_a|$ (so that $1-\eta = |\mathcal{F}_a^{(2)}|/|\mathcal{F}_a|$), and (ii) the mean valuations of flights in $\mathcal{F}_a^{(1)}$, that is, $\mu_1$ (such that $\eta\mu_1 + (1-\eta)\mu_2 = 1$). Within each set, $\mathcal{F}_a^{(1)}$ and $\mathcal{F}_a^{(2)}$, we sort flights from the least valuable to the most valuable using 10 random permutations. In other words, the 10 samples have the same sets of flight valuations, but differ in terms of which actual flights are more flexible and which are less flexible.

Table 5 shows results (with $A_{\text{MAX}} = 11$ and $D_{\text{MAX}} = 15$, which are the most stringent set of on-time performance targets from those in Table 3) under different sets of flight valuations provided by DAL and AAL—similar results are obtained by varying the flight valuations provided by the other airlines. The first row provides a baseline where all flights are equally valued (i.e., $v_i = 1$ for all $i \in \mathcal{F}$). In the top half, we assume that $\mathcal{F}_a^{(1)}$ and $\mathcal{F}_a^{(2)}$ both comprise 50% of the flights from DAL or AAL, and we progressively increase the valuation differential $\mu_2 - \mu_1$. In the bottom half, we fix $\mu_1 = 0.75$ and we progressively decrease the proportion of flights in $\mathcal{F}_a^{(1)}$ (and we thus decrease $\mu_2$ to ensure that $\eta\mu_1 + (1-\eta)\mu_2 = 1$). Table 5 reports, in each scenario, the total schedule displacement $\sum_{i\in\mathcal{F}_a} |u_i|$ of *each* airline $a$ obtained in the EQMAX solution (Problem $\widehat{\text{P3}(\rho*)}$), as well as the prices of equity and efficiency, averaged across all 10 samples.

The observations from variations in $\mu_2 - \mu_1$ (top) and in $\eta$ (bottom) are threefold. First, as an airline's flight valuations become more differentiated, the displacement of this airline's schedule increases. In turn, flight valuations create, for each airline, a trade-off between prioritizing *which* flights get rescheduled, on one hand, and minimizing their total displacement, on the other hand. Second, as the variance in any airline's flight valuations increases, other airlines' displacements do not change significantly (in fact, sometimes they decrease a little). In other words, the model can account for any airline's scheduling preferences without negatively impacting the other airlines. Third, the price of equity is much smaller than the price of efficiency across all of the scenarios considered, therefore indicating strong gains in interairline equity at small efficiency losses.

We further investigate the trade-off between equity and efficiency in the case where average flight valuations may vary across the airlines. For each flight $i \in \mathcal{F}$, we estimate $v_i$ as the product of the aircraft size and the average "nonstop" airfare. This aims to capture variations in operating revenues across flights. Aircraft sizes (i.e., number of seats) are obtained from the ASPM database (Federal Aviation

**Table 5.** Results for September 18, 2007 Under Differentiated Flight Valuations for DAL and AAL

| Scenario | | | For variations in DAL's flight valuations | | | | | | For variations in AAL's flight valuations | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mu_1$ | $\mu_2$ | $\eta$ (%) | DAL | AAL | JBU | Others | $P_{\text{eq}}$ (%) | $P_{\text{eff}}$ (%) | DAL | AAL | JBU | Others | $P_{\text{eq}}$ (%) | $P_{\text{eff}}$ (%) |
| 1.0 | 1.0 | — | 57.0 | 46.0 | 31.0 | 72.0 | 0 | 0 | 57.0 | 46.0 | 31.0 | 72.0 | 0 | 0 |
| 0.9 | 1.1 | 50 | 57.7 | 44.0 | 36.2 | 70.0 | 4.4 | 15.4 | 55.5 | 46.9 | 36.0 | 69.5 | 2.8 | 24.8 |
| 0.8 | 1.2 | 50 | 57.0 | 44.7 | 36.4 | 70.5 | 4.5 | 15.3 | 53.4 | 50.0 | 34.6 | 67.9 | 2.4 | 26.5 |
| 0.7 | 1.3 | 50 | 58.1 | 44.1 | 35.9 | 70.2 | 5.6 | 14.5 | 52.2 | 50.5 | 33.1 | 66.6 | 2.0 | 23.3 |
| 0.6 | 1.4 | 50 | 59.8 | 43.5 | 36.2 | 69.1 | 6.8 | 14.4 | 51.5 | 59.1 | 32.7 | 65.1 | 2.1 | 15.8 |
| 0.5 | 1.5 | 50 | 63.5 | 43.4 | 35.3 | 68.8 | 7.2 | 13.7 | 49.9 | 64.0 | 32.1 | 63.4 | 2.3 | 11.6 |
| 0.4 | 1.6 | 50 | 67.3 | 43.5 | 34.9 | 68.9 | 8.1 | 13.4 | 48.8 | 70.7 | 31.4 | 61.4 | 2.0 | 7.8 |
| 0.3 | 1.7 | 50 | 73.2 | 42.3 | 34.0 | 67.1 | 9.3 | 13.3 | 46.5 | 78.4 | 31.7 | 59.0 | 2.7 | 9.7 |
| 0.2 | 1.8 | 50 | 77.4 | 42.4 | 34.8 | 67.2 | 11.2 | 13.4 | 44.4 | 87.5 | 29.9 | 56.5 | 2.8 | 10.7 |
| 0.1 | 1.9 | 50 | 84.1 | 42.9 | 34.0 | 67.8 | 13.5 | 11.8 | 42.0 | 95.8 | 29.6 | 53.4 | 3.8 | 15.7 |
| 0.75 | 3.25 | 90 | 57.6 | 44.4 | 36.6 | 70.4 | 4.1 | 16.9 | 49.3 | 42.3 | 32.2 | 62.4 | 2.4 | 19.9 |
| 0.75 | 2.00 | 80 | 57.4 | 44.7 | 36.6 | 70.7 | 4.4 | 17.3 | 54.9 | 48.7 | 35.2 | 69.0 | 2.2 | 19.6 |
| 0.75 | 1.58 | 70 | 57.3 | 44.4 | 36.3 | 70.4 | 4.3 | 17.2 | 53.5 | 49.6 | 35.1 | 68.4 | 2.2 | 17.9 |
| 0.75 | 1.37 | 60 | 57.2 | 44.7 | 36.2 | 70.7 | 4.5 | 16.4 | 54.3 | 50.8 | 35.1 | 68.2 | 2.6 | 17.7 |
| 0.75 | 1.25 | 50 | 57.5 | 44.1 | 36.4 | 69.7 | 5.1 | 15.2 | 53.0 | 52.0 | 33.8 | 67.6 | 1.9 | 21.2 |
| 0.75 | 1.17 | 40 | 57.6 | 44.3 | 36.0 | 70.5 | 5.5 | 13.6 | 52.7 | 54.2 | 34.4 | 66.9 | 2.7 | 24.2 |
| 0.75 | 1.11 | 30 | 58.6 | 44.1 | 36.5 | 70.1 | 6.2 | 12.0 | 51.9 | 55.8 | 33.4 | 65.9 | 2.8 | 33.0 |
| 0.75 | 1.06 | 20 | 59.7 | 44.0 | 35.6 | 69.6 | 6.6 | 10.8 | 51.0 | 58.2 | 33.0 | 65.0 | 2.8 | 35.8 |
| 0.75 | 1.03 | 10 | 66.6 | 43.1 | 35.4 | 67.9 | 6.9 | 9.9 | 50.0 | 62.6 | 31.8 | 63.3 | 2.7 | 33.8 |

Administration 2013). Nonstop airfares are obtained from the Bureau of Transportation Statistics website for each domestic origin-destination market (averaged over the third quarter of 2007) (Bureau of Transportation Statistics 2013). International airfares are inferred by using a regression model to estimate airfares as a function of flight distance (calibrated using domestic markets data), to which we added a 20% premium, reflecting the fact that international flights are typically more expensive than domestic flights traveling the same distance. Of course, this procedure can only be treated as highly approximate, and many other drivers of operating profitability and other measures of scheduling flexibility (e.g., load factors, operating margins, connecting passengers) could not be estimated from the publicly available data. Our aim here is to assess the impact of such differentiated flight valuations on the trade-off between efficiency and equity in scheduling interventions.

Table 6 shows the results for different on-time performance targets $A_{\text{MAX}}$ and $D_{\text{MAX}}$. For each set of values for these targets, we report the largest flight displacement (i.e., $\delta^* = \max_{i\in\mathcal{F}} |u_i|$), the total schedule displacement (i.e., $\Delta_0 = \sum_{i\in\mathcal{F}} |u_i|$), the total weighted schedule displacement (i.e., $\Delta^* = \sum_{i\in\mathcal{F}} v_i|u_i|$), and the disutility of each airline (i.e., the average per-flight weighted displacement $\sigma_a^* = (1/|\mathcal{F}_a|)\sum_{i\in\mathcal{F}_a} v_i|u_i|$ for each airline $a$ under the EFFMAX-EQMIN (Problem $\overline{\text{P2}}$), EFFMAX-EQMAX (Problem $\widehat{\text{P3}(0)}$), and EQMAX (Problem $\widehat{\text{P3}(\rho*)}$) solutions. Note that $\Delta^*$ and $\sigma_a^*$ are expressed in dollars, since our flight valuations are based on revenue estimates). We also report the price of equity and the price of efficiency. First, note that the EFFMAX-EQMIN solution and the EFFMAX-EQMAX solution result in the same airline disutilities. This contrasts with the case of uniform valuations (Section 5.2), where the EFFMAX-EQMAX solution vastly improved interairline equity. This is because differentiations in flight valuations restrict the set of efficiency-maximizing solutions, thus reducing the flexibility to select the set of flights to be rescheduled in an equitable way. Second, the EQMAX solution balances per-flight weighted displacement much more equitably across the four groups of airlines. Third, interairline equity is achieved through moderate increases in the number of flights displaced and, in some instances, by rescheduling *fewer* flights than under the EFFMAX-EQMAX solution. Last, even though the price of equity is higher than in our previous tests, the price of efficiency remains significantly higher than the price of equity. This is particularly true for the more aggressive on-time performance targets, which can be explained by the fact that the flexibility to select the set of flights to displaced in a more equitable manner increases with the schedule displacement. Overall, these results suggest that interairline equity can be achieved through comparatively small increases in efficiency, even under strong differentiations in flight valuations across the airlines and across the flights of an airline. Note that such differentiations could arise in many future extensions of the existing mechanisms for airport demand management.

## 6. Conclusion

Any airport demand management scheme involves a trade-off between mitigating airport congestion, on one hand, and minimizing interference with airlines' competitive scheduling, on the other hand. In this paper, we have developed, optimized, and assessed models for airport scheduling interventions that, for the first time, incorporate interairline equity considerations. The resulting Integrated Capacity Utilization and Scheduling Model with Equity Considerations (ICUSM-E) relies on an original multilevel modeling

**Table 6.** Results for September 18, 2007 Under Revenue-Based Valuations, for Different On-Time Performance Targets

| Targets | | | Displacement | | | Airline disutilities | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $A_{\text{MAX}}$ | $D_{\text{MAX}}$ | Solution | $\delta^*$ | $\Delta_0$ | $\Delta^*$ ($) | DAL ($) | AAL ($) | JBU ($) | Others ($) | Performance |
| 14 | 23 | $\overline{\text{P2}}$ | 1 | 23 | 257,000 | 119 | 432 | 101 | 218 | $P_{\text{eq}} = 20.2\%$ |
| | | $\widehat{\text{P3}(0)}$ | 1 | 23 | 257,000 | 119 | 432 | 101 | 218 | $P_{\text{eff}} = 54.1\%$ |
| | | $\widehat{\text{P3}(\rho*)}$ | 1 | 26 | 308,000 | 267 | 280 | 183 | 218 | |
| 13 | 20 | $\overline{\text{P2}}$ | 1 | 41 | 492,000 | 368 | 781 | 384 | 257 | $P_{\text{eq}} = 17.8\%$ |
| | | $\widehat{\text{P3}(0)}$ | 1 | 41 | 492,000 | 368 | 781 | 384 | 257 | $P_{\text{eff}} = 53.4\%$ |
| | | $\widehat{\text{P3}(\rho*)}$ | 1 | 44 | 580,000 | 455 | 509 | 420 | 489 | |
| 12 | 18 | $\overline{\text{P2}}$ | 1 | 75 | 1,200,000 | 809 | 1,566 | 1,596 | 611 | $P_{\text{eq}} = 17.6\%$ |
| | | $\widehat{\text{P3}(0)}$ | 1 | 75 | 1,200,000 | 809 | 1,566 | 1,596 | 611 | $P_{\text{eff}} = 33.6\%$ |
| | | $\widehat{\text{P3}(\rho*)}$ | 1 | 85 | 1,410,000 | 1,186 | 1,189 | 1,173 | 1,197 | |
| 11 | 15 | $\overline{\text{P2}}$ | 1 | 226 | 5,163,000 | 4,389 | 6,530 | 5,909 | 2,393 | $P_{\text{eq}} = 6.9\%$ |
| | | $\widehat{\text{P3}(0)}$ | 1 | 226 | 5,163,000 | 4,389 | 6,530 | 5,909 | 2,393 | $P_{\text{eff}} = 38.5\%$ |
| | | $\widehat{\text{P3}(\rho*)}$ | 1 | 224 | 5,518,000 | 4,714 | 4,744 | 4,686 | 4,686 | |

architecture that optimizes scheduling interventions based on on-time performance, efficiency, and interairline equity objectives.

Theoretical results have shown that, in the absence of network connections and under the standard paradigm that "a flight is a flight" (i.e., all flights are equally inconvenient to reschedule), efficiency and interairline equity can be jointly maximized if the interventions involve accepting or rejecting flight requests without changing the timetabling of the flights, or if the interventions only involve temporal shifts in demand and some additional scheduling conditions are satisfied. Computational results suggested that, under a wide range of realistic and hypothetical scenarios, interairline equity can be achieved at small efficiency losses (if any). In other words, achieving maximum equity requires no (or small) sacrifice in terms of efficiency losses. On the other hand, for some of our computational scenarios, our results showed that ignoring interairline equity (i.e., considering efficiency-based objectives exclusively, or, in some cases, requiring maximum efficiency) may lead to highly inequitable outcomes. This further highlights that it is critical to explicitly incorporate interairline equity objectives in the optimization of scheduling interventions. In turn, this offers the potential to extend existing approaches to airport demand management (either the slot control policies in place at busy airports outside the United States, or the scheduling practices at a few of the busiest U.S. airports where flight caps are in place) in a way that balances scheduling interventions fairly among the airlines, thus considerably enhancing their applicability in practice.

The potential equity benefits of scheduling interventions also motivate future research directions on airport scheduling interventions. Most importantly, this paper has assumed knowledge of the scheduling inputs provided by the airlines. An important opportunity lies in the design and optimization of scheduling intervention mechanisms through which the airlines can provide their preferred schedules of flights (and, potentially, some other inputs as well). The design of such mechanisms would also create an opportunity to analyze the strategic interactions among the airlines and minimize the potential for gaming. More broadly, this research lays down the modeling framework to optimize nonmonetary mechanisms and compare them to market-based mechanisms such as congestion pricing and slot auctions, based on common on-time performance, efficiency, and interairline equity objectives. The approach developed in this paper provides the methodological foundation to address such problems of airport capacity allocation to mitigate delay externalities, promote airline competition, and maximize social welfare in a way, as the results have shown, that ensures interairline equity.

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